CHAPTER 5

Database Design (DBD) Toolkit

This chapter introduces a toolkit written in SWI-Prolog (a freely available Prolog system for the Unix as well as the Windows operating systems) that allows the student to learn and test out the various concepts, definitions, and algorithms associated with relational database design using functional dependency theory.

The DBD system is available as SWI-Prolog source code in a single file, dbd.pl. This file needs to be consulted (read into the Prolog interpreter's memory) before invoking any of the DBD predicates.

5.1 Coding Relational Schemas and Functional Dependencies

The DBD system uses lists to code the relational schemas and functional dependencies. Variables in Prolog begin with an upper-case letter and hence attributes of relational schemes are coded as Prolog constants – terms that begin with a lower-case letter. Consider the following relational schema and associated set of functional dependencies:

R(A, B, C, D, E)F = { A \rightarrow B, AC \rightarrow D, E \rightarrow AB, AD \rightarrow E, ABC \rightarrow D }

These will be coded as the following Prolog predicates in DBD:

5.2 Invoking the SWI-Prolog Interpreter

Once the SWI-Prolog system is installed in your server or local computer, the interpreter is invoked using the following terminal command:

\$pl

Here \$ is the command prompt. In the rest of the chapter, all user inputs are shown in **bold** font and system displays in regular font.

After printing a welcome message, the interpreter responds with the following prompt:

?-

At this prompt, the user may enter a Prolog query to invoke a predicate. In order to invoke one of the predicates of the DBD system, the user must first load Prolog interpreter's memory with the DBD program using the following command/query:

?- ['dbd.pl'].

Note that all commands/queries to Prolog end with a period (".") symbol. To exit the Prolog system, the halt command/query is entered at the Prolog prompt as follows:

?- halt.

To solve a particular problem using the DBD system, the student will usually create a file containing the code to represent the relational schema and the functional dependencies. In addition, one or more Prolog rules may be included to execute predicates to solve the problem at hand. For example, consider the problem of finding one or more candidate keys for the relational schema and functional dependencies mentioned earlier in the section. The student will create a text file with the following code:

The first two predicates (schema and fds) code the relational schema and functional dependencies. The third rule invokes the candkey predicate of the DBD system to find the candidate keys for R and F. The answer is captured in the answer predicate with one argument. Let us assume that the above code is present in a file called example.pl. The following interaction with the SWI-Prolog system shows the steps necessary to find the answer(s) to the problem:

\$ pl Welcome to SWI-Prolog (Multi-threaded, Version 5.4.7) Copyright (c) 1990-2003 University of Amsterdam. SWI-Prolog comes with ABSOLUTELY NO WARRANTY. This is free software, and you are welcome to redistribute it under certain conditions. Please visit http://www.swi-prolog.org for details. For help, use ?- help(Topic). or ?- apropos(Word). ?- ['dbd.pl']. % dbd.pl compiled 0.02 sec, 30,740 bytes Yes ?- ['example.pl']. % example.pl compiled 0.00 sec, 1,116 bytes Yes ?- answer(K). K = [c, e] ;

```
K = [a, c] ;
No
?- answer(K).
K = [c, e]
Yes
?- halt.
s
```

The user inputs are shown in **bold** font. After invoking the interpreter, the user first loads the DBD system code (dbd.pl). The user then loads the code to solve the problem at hand (example.pl). Finally, the user submits the query, **answer(K)**, to determine the candidate keys. By default, every Prolog system responds with the first answer to the query and waits for user input. At this point the user has two choices: press the ENTER key to stop looking at further answers to the query or press the semi-colon key to request Prolog to display the next answer to the query. In the above screen capture, both options are shown.

5.3 DBD System Predicates

The DBD system predicates are discussed in this section. Almost all of the predicates take as input the relation schema R and a set of functional dependencies F. R and F must be coded properly as indicated in earlier sections. In particular, the attribute names must begin with a lower-case letter. The predicate names themselves also must begin with a lower-case letter.

5.3.1 xplus(R,F,X,Xplus)

The xplus predicate takes as input a relation schema R, a set of functional dependencies F, and a subset X of R and produces as output the attribute-closure of X under F, i.e. the set of attributes that are functionally dependent on X. The following shows a sample interaction with the Prolog system to compute the attribute closure of $\{A, C\}$ for the relational schema and functional dependencies mentioned earlier in this chapter (the file e2.pl is assumed to contain the schema and fds predicates):

```
?- ['dbd.pl'].
% dbd.pl compiled 0.02 sec, 30,740 bytes
Yes
?- ['e2.pl'].
% e2.pl compiled 0.00 sec, 1,096 bytes
Yes
?- schema(R),fds(F),xplus(R,F,[a,c],Xplus).
R = [a, b, c, d, e]
F = [[[a], [b]], [[a, c], [d]], [[e], [a, b]], [[a, d], [e]], [[a, b, c], [d]]]
Xplus = [a, b, c, d, e]
```

```
Yes
?- halt.
$
```

5.3.2 finfplus(R,F,[X,Y])

The finfplus predicate takes as input a relation schema R, a set of functional dependencies F, and a functional dependency $X \rightarrow Y$ (in list coded form [X, Y]) and returns "yes" if $X \rightarrow Y$ is in the closure of F and "no" otherwise. An example using the same R and F as before follows:

```
?- ['dbd.pl'].
% dbd.pl compiled 0.02 sec, 30,740 bytes
Yes
?- ['e2.pl'].
% e2.pl compiled 0.00 sec, 1,096 bytes
Yes
?- schema(R), fds(F), finfplus(R,F,[[a,b],[c]]).
No
?- schema(R), fds(F), finfplus(R,F,[[a,d],[b]]).
R = [a, b, c, d, e]
F = [[[a], [b]], [[a, c], [d]], [[e], [a, b]], [[a, d], [e]], [[a, b, c], [d]]]
Yes
```

In the above screen capture we observe that the functional dependency $AB \rightarrow C$ is not in the closure of F whereas the functional dependency $AD \rightarrow B$ is in the closure of F.

5.3.3 fplus(R,F,Fplus)

The fplus predicate takes as input a relational schema R and a set of functional dependencies F and returns as output the closure of F in the third parameter. Typically the number of functional dependencies in the closure of F is large and in the worst case can be exponential in the number of attributes of R.

Consider the following Prolog code in file e3.pl:

This program adds the display rules to the e2.pl file and invokes the displayFDs predicate on the closure of F. All functional dependencies that are implied by F are displayed by the following invocation:

```
?- ['dbd.pl'].
% dbd.pl compiled 0.02 sec, 30,740 bytes
Yes
?- ['e3.pl'].
% e3.pl compiled 0.01 sec, 1,700 bytes
Yes
?- go.
[a]-->[a, b]
[a]-->[b]
...
...
```

Note: The fplus predicate does not generate any trivial functional dependencies (dependencies whose RHS attributes are also present in the LHS attributes).

5.3.4 implies(R,F1,F2) and equiv(R,F1,F2)

The implies predicate takes as input a relational schema R and two sets of functional dependencies F1 and F2. It returns "yes" if F1 implies F2, i.e. if each functional dependency in F2 is in the closure of F1; returns "no" otherwise.

The equiv predicate takes as input a relational schema R and two sets of functional dependencies F1 and F2. It returns "yes" if F1 is equivalent to F2, i.e. if each functional dependency in F2 is in the closure of F1 and if each functional dependency in F1 is in the closure of F2; returns "no" otherwise.

Consider the relational schema R(A, B, C) and the two sets of functional dependencies:

F1 = {A \rightarrow BC, B \rightarrow A} F2 = {A \rightarrow B, A \rightarrow C}

The above schema and functional dependencies can be coded into a file e4.pl as follows:

```
schema([a,b,c]).
fds1([[[a],[b,c]], [[b],[a]]]).
fds2([[[a],[b]], [[a],[c]]]).
```

The following screen capture illustrates the implies and equiv predicates:

```
?- ['dbd.pl'].
% dbd.pl compiled 0.02 sec, 30,740 bytes
```

```
Yes
?- ['e4.pl'].
% e4.pl compiled 0.00 sec, 1,120 bytes
Yes
?- schema(R), fds1(F1), fds2(F2), implies(R,F1,F2).
R = [a, b, c]
F1 = [[[a], [b, c]], [[b], [a]]]
F2 = [[[a], [b]], [[a], [c]]]
Yes
?- schema(R), fds1(F1), fds2(F2), implies(R,F2,F1).
No
?- schema(R), fds1(F1), fds2(F2), equiv(R,F1,F2).
No
?-
```

5.3.5 superkey(R,F,K) and candkey(R,F,K)

The superkey and candkey predicates both take a relation schema R and a set of functional dependencies as input in the first two parameters. The third parameter may be a constant (i.e. an instantiated list of attributes of R) or a variable.

In case the third parameter is instantiated to a list of attributes the superkey (candkey) predicate with return "yes" if the instantiated third parameter is a super key (candidate key) of R; "no" otherwise.

In case the third parameter is a variable, the predicate superkey (candkey) will generate all super keys (candidate keys) one at a time.

Consider the following e5.pl file containing a relation schema and associated functional dependencies:

```
schema([a,b,c]).
fds([[[a],[b,c]], [[b],[a]]]).
```

The following screen capture illustrates usage of the superkey and candkey predicates:

```
?- ['dbd.pl'].
% dbd.pl compiled 0.01 sec, 30,428 bytes
Yes
?- ['e5.pl'].
% e5.pl compiled 0.00 sec, 1,124 bytes
Yes
?- schema(R), fds(F), candkey(R,F,K).
```

```
R = [a, b, c]
F = [[[a], [b, c]], [[b], [a]]]
K = [b];
R = [a, b, c]
F = [[[a], [b, c]], [[b], [a]]]
K = [a];
No
?- schema(R), fds(F), superkey(R,F,K).
R = [a, b, c]
F = [[[a], [b, c]], [[b], [a]]]
K = [a, b, c];
R = [a, b, c]
F = [[[a], [b, c]], [[b], [a]]]
K = [b, c];
R = [a, b, c]
F = [[[a], [b, c]], [[b], [a]]]
K = [b];
R = [a, b, c]
F = [[[a], [b, c]], [[b], [a]]]
K = [a, c];
R = [a, b, c]
F = [[[a], [b, c]], [[b], [a]]]
K = [a];
R = [a, b, c]
F = [[[a], [b, c]], [[b], [a]]]
K = [a, b];
No
?- schema(R), fds(F), candkey(R,F,[a,b]).
No
?- schema(R), fds(F), superkey(R,F,[a,b]).
R = [a, b, c]
F = [[[a], [b, c]], [[b], [a]]]
Yes
?-
```

5.3.6 mincover(R,F,FC)

The mincover predicate takes as input a relation schema R and a set of functional dependencies F and returns as output in the third parameter the minimal cover for F.

Consider the following relation schema and functional dependencies in file e6.pl:

schema([a,b,c,d,e]).
fds([[[a],[b,c]], [[b],[a]], [[a,b],[d]], [[a],[d]], [[b,c],[a]]]).

The functional dependencies coded in the file are:

 $F = \{A \rightarrow BC, B \rightarrow A, AB \rightarrow D, A \rightarrow D, BC \rightarrow A\}$

The following screen capture illustrates the mincover predicate usage:

```
?- ['dbd.pl'].
% dbd.pl compiled 0.02 sec, 30,740 bytes
Yes
?- ['e6.pl'].
% e6.pl compiled 0.00 sec, 1,072 bytes
Yes
?- schema(R),fds(F),mincover(R,F,MinF).
R = [a, b, c, d, e]
F = [[[a], [b, c]], [[b], [a]], [[a, b], [d]], [[a], [d]], [[b, c], [a]]]
MinF = [[[a], [b, c, d]], [[b], [a]]]
Yes
?-
```

The minimum cover for the set of functional dependencies is

 $MinF = \{A \rightarrow BCD, B \rightarrow A\}.$

5.3.7 ljd(R,F,R1,R2), ljd(R,F,D), and fpd(R,F,D)

The predicates discussed in this sub-section are related to decompositions of relation schemas. We represent decompositions using lists of relational schemas. For example, the decomposition $D = \{ABC, ABD, BDE\}$ of R = ABCDE is represented as the following Prolog term:

D = [[a,b,c],[a,b,d],[b,d,e]]

Testing for lossless join decomposition

There are two versions of the ljd predicate.

The first version of the ljd predicate takes as input a relation scheme R, a set of functional dependencies F, and two relation schemas R1 and R2 that are decompositions of R. The predicate returns "yes" if (R1, R2) is a lossless join decomposition of R; "no" otherwise.

As an example, consider the R = ABC and $F = \{A \rightarrow B\}$. Consider the decomposition of R into two sub-schemas R1 = AB and R2 = AC. The following screen capture illustrates the use of the first version of ljd predicate:

```
?- ['dbd.pl'].
% dbd.pl compiled 0.02 sec, 30,740 bytes
Yes
?- ljd([a,b,c],[[[a],[b]]],[a,b],[a,c]).
Yes
?-
```

The second version of the ljd predicate takes as input a relation scheme R, a set of functional dependencies F, and a decomposition D into two or more sub-schemas. D is represented as a list of relation schemas where each schema is itself a list of attributes. The predicate returns "yes" if D is a lossless join decomposition of R; "no" otherwise. This version implements the "matrix" method (Algorithm 11.1 of the Elmasri/Navathe text) to test for lossless join and prints the final matrix.

As an example, consider the following file (named e7a.pl) containing a relational schema, a set of functional dependencies and a decomposition into more than two sub-schemas:

```
schema([a,b,c,d,e]).
fds([[[a],[c]], [[b],[c]], [[c],[d]], [[d,e],[c]], [[c,e],[a]]]).
decomp([[a,d],[a,b],[b,e],[c,d,e],[a,e]]).
```

The following screen capture illustrates the usage of the more general version of the ljd predicate:

```
?- ['dbd.pl'].
% dbd.pl compiled 0.02 sec, 30,740 bytes
Yes
?- ['e7a.pl'].
% e7a.pl compiled 0.01 sec, 1,440 bytes
Yes
?- schema(R), fds(F), decomp(D), ljd(R,F,D).
[[a, 1], [b, 1, 2], [a, 3], [a, 4], [b, 1, 5]]
[[a, 1], [a, 2], [a, 3], [a, 4], [b, 2, 5]]
[[a, 1], [a, 2], [a, 3], [a, 4], [a, 5]]
[[a, 1], [b, 4, 2], [a, 3], [a, 4], [a, 5]]
[[a, 1], [b, 5, 2], [a, 3], [a, 4], [a, 5]]
R = [a, b, c, d, e]
F = [[[a], [c]], [[b], [c]], [[c], [d]], [[d, e], [c]], [[c, e], [a]]]
D = [[a, d], [a, b], [b, e], [c, d, e], [a, e]]
Yes
?-
```

Note that the ljd predicate execution prints the final matrix to the screen; In this case it can be observed that the third row is turned into an "all-a" row.

Testing for FD-preserving decomposition

The fpd predicate takes as input a relation scheme R, a set of functional dependencies F, and a decomposition D into two or more sub-schemas. D is represented as a list of relation schemas where each schema is itself a list of attributes. The predicate returns "yes" if D is a functional dependency preserving decomposition of R; "no" otherwise.

As an example, consider the following file (named e7b.pl) containing a relational schema, a set of functional dependencies and a decomposition into sub-schemas:

```
schema([a,b,c,d]).
fds([[[a],[b]], [[b],[c]], [[c],[d]], [[d],[a]]]).
decomp([[a,b],[b,c],[c,d]]).
```

The following screen capture illustrates the usage of the fpd predicate:

```
?- ['dbd.pl'].
% dbd.pl compiled 0.03 sec, 30,740 bytes
Yes
?- ['e7b.pl'].
% e7b.pl compiled 0.00 sec, 1,260 bytes
Yes
?- schema(R), fds(F), decomp(D), fpd(R,F,D).
Considering [a]-->[b]
Xplus=[a, b, c, d]
Considering [b]-->[c]
Xplus=[a, b, c, d]
Considering [c]-->[d]
Xplus=[a, b, c, d]
Considering [d]-->[a]
Xplus=[a, b, c, d]
R = [a, b, c, d]
F = [[[a], [b]], [[b], [c]], [[c], [d]], [[d], [a]]]
D = [[a, b], [b, c], [c, d]]
Yes
?-
```

Note that the fpd predicate execution prints each functional dependency in F and displays the attribute closure of the LHS of the functional dependency with respect to the "projected" functional dependencies. The algorithm implemented in this predicate is from "*Jeffrey D. Ullman, Principles of Database Systems, Second Edition, Computer Science Press, 1982*".

5.3.8 is3NF(R,F) and threenf(R,F,D)

The is3NF predicate takes as input a relation schema R and a set of associated functional dependencies and returns "yes" if the relation schema is in 3NF; "no" otherwise.

The threenf predicate takes as input a relation schema R and a set of associated functional dependencies and returns in the third parameter a decomposition of R into 3NF. The threenf predicate implements Algorithm 11.4 of the Elmasri/Navathe text.

As an example, consider the following file (named e8.pl) containing a relational schema and a set of functional dependencies:

```
schema([a,b,c,d]).
fds([[[a,b],[c]],[[b],[d]],[[b,c],[a]]]).
```

The following screen capture illustrates the 3NF test as well as a 3NF decomposition of the relation schema.

```
?- ['dbd.pl'].
% dbd.pl compiled 0.02 sec, 30,740 bytes
Yes
?- ['e8.pl'].
% e8a.pl compiled 0.01 sec, 932 bytes
Yes
?- schema(R), fds(F), is3NF(R,F).
No
?- schema(R), fds(F), threenf(R,F,R3NF).
R = [a, b, c, d]
F = [[[a, b], [c]], [[b], [d]], [[b, c], [a]]]
R3NF = [[a, b, c], [b, d]]
Yes
?-
```

As can be seen in the above interaction with the DBD system, the schema R = ABCD is not in 3NF and is decomposed into 3NF sub-schemas ABC and BD.

5.3.9 isBCNF(R,F) and bcnf(R,F,D)

The isBCNF predicate takes as input a relation schema R and a set of associated functional dependencies and returns "yes" if the relation schema is in BCNF; "no" otherwise.

The bcnf predicate takes as input a relation schema R and a set of associated functional dependencies and returns in the third parameter a decomposition of R into BCNF. The bcnf predicate implements Algorithm 11.3 of the Elmasri/Navathe text.

As an example, consider the same e8.pl file used in the previous sub-section.

The following screen capture illustrates the BCNF test as well as a BCNF decomposition of the relation schema.

```
?- ['dbd.pl'].
% dbd.pl compiled 0.02 sec, 30,740 bytes
Yes
?- ['e8.pl'].
% e8.pl compiled 0.00 sec, 928 bytes
Yes
?- schema(R), fds(F), isBCNF(R,F).
No
?- schema(R), fds(F), bcnf(R,F,D).
Scheme to decompose = [a, b, c, d] Offending FD: [b]-->[d]
Final Result is:
[a, b, c]
[b, d]
R = [a, b, c, d]
F = [[[a, b], [c]], [[b], [d]], [[b, c], [a]]]
D = [[a, b, c], [b, d]];
```

As can be seen in the above interaction with the DBD system, the schema R = ABCD is not in BCNF and is decomposed into BCNF sub-schemas ABC and BD. Note that the bcnf predicate implementation also prints out the schema that is being decomposed along with the "offending" functional dependency.

Exercises

For all the problems in this set of exercises, use the predicates in the DBD system to answer the questions.

Chapter 10 (Elmasri/Navathe Text) Problems:

1. Consider the following two sets of functional dependencies:

 $F = \{A \rightarrow C, AC \rightarrow D, E \rightarrow AD, E \rightarrow H\}$ G = {A $\rightarrow CD, E \rightarrow AH$ }

Check whether they are equivalent.

2. Consider the relation schema

EMP DEPT(ename,ssn,bdate,address,dnumber,dname,dmgrssn)

and the following set G of functional dependencies on EMP DEPT:

```
ssn \rightarrow ename, bdate, address, dnumber dnumber \rightarrow dname, dmgrssn
```

Calculate the closures $\{ssn\}^+$ and $\{dnumber\}^+$.

- 3. Is the set of functional dependencies G in Exercise 2 minimal? If not, find a minimal set of functional dependencies that is equivalent to G.
- 4. Consider the universal relation R = { A, B, C, D, E, F, G, H, I} and the set of functional dependencies:

 $F = \{AB \rightarrow C, A \rightarrow DE, B \rightarrow F, F \rightarrow GH, D \rightarrow IJ\}$

What is the key for R? Decompose R into 3NF relations.

5. Repeat Exercise 4 for the same R but a different set of functional dependencies

 $G = \{AB \rightarrow C, BD \rightarrow EF, B \rightarrow F, F \rightarrow GH, D \rightarrow IJ\}$

6. Consider a relation R(A,B,C,D,E) with the following functional dependencies:

 $F = \{AB \rightarrow C, CD \rightarrow E, DE \rightarrow B\}$

Is AB a candidate key of this relation? If not, is ABD?

 Consider the relation R, which has attributes that hold schedules of courses and sections at a university;

Suppose the following functional dependencies hold on R:

```
roomno,dayshours,semester,year →
instructorssn,courseno,secno
```

Find the candidate keys for R and decompose the relation into 3NF relations.

Chapter 11 (Elmasri/Navathe Text) Problems:

8. Consider the relation schema

LOTS (propertyid, countyname, lotno, area, price, taxrate)

and associated set of functional dependencies:

propertyid → countyname,lotno,area,price,taxrate
countyname,lotno → propertyid,area,price,taxrate
countyname → taxrate
area → price

Determine if the decomposition of LOTS into

```
LOTS1AX (propertyis, area, lotno)
LOTS1AY (area, countyname)
LOTS1B (area, price)
LOTS2 (countyname, taxrate)
```

is a lossless join decomposition?

9. Consider the universal relation R = { A, B, C, D, E, F, G, H, I} and the set of functional dependencies:

 $F = \{AB \rightarrow C, A \rightarrow DE, B \rightarrow F, F \rightarrow GH, D \rightarrow IJ\}$

Determine the candidate keys for R, find a minimal cover for F, and decompose R into 3NF relations.

10. Consider the universal relation R = { A, B, C, D, E, F, G, H, I} and the set of functional dependencies:

 $G = \{AB \rightarrow C, BD \rightarrow EF, B \rightarrow F, F \rightarrow GH, D \rightarrow IJ\}$

Determine the candidate keys for R, find a minimal cover for F, and decompose R into 3NF relations.

11. Consider the universal relation R = { A, B, C, D, E, F, G, H, I} and the set of functional dependencies:

 $F = \{AB \rightarrow C, A \rightarrow DE, B \rightarrow F, F \rightarrow GH, D \rightarrow IJ\}$

Decompose R into BCNF relations. Repeat the exercise for

 $G = \{AB \rightarrow C, BD \rightarrow EF, B \rightarrow F, F \rightarrow GH, D \rightarrow IJ\}$

12. Consider a relation R(A,B,C,D,E) with the following functional dependencies:

 $F = \{AB \rightarrow C, CD \rightarrow E, DE \rightarrow B\}$

Determine the candidate keys for R and decompose R into 3NF relations and into BCNF relations.

Repeat the exercise for

and F containing the following functional dependencies:

```
courseno → offeringdept,credithours,courselevel,
courseno,secno,semester,year →
dayshours,roomno,noofstudents,instructorssn
```

```
roomno,dayshours,semester,year → instructorssn,courseno,secno
```

13. Consider the following decompositions for the relation schema

R(a,b,c,d,e,f,g,h,i)

and the set of associated functional dependencies:

 $F = \{ab \rightarrow c, a \rightarrow de, b \rightarrow f, f \rightarrow gh, d \rightarrow ij\}.$

Determine whether each of the following decompositions has the (a) dependency preservation property, and (b) the lossless join property with respect to F:

| i. | D1 = {R1,R2,R3,R4,R5}; R1 = {a,b,c}, R2 = {a,d,e}, R3 = {b,f}, R4 = {f,g,h}, R5 = {d,i,j}. |
|------|--|
| ii. | D2 = {R1,R2,R3}; R1 = {a,b,c,d,e}, R2 = {b,f,g,h}, R3 = d,i,j} |
| iii. | D3 = {R1,R2,R3,R4,R5}; R1 = {a,b,c,d}, R2 = {d,e}, R3 = {b,f}, R4 = {f,g,h}, R5 = {d,i,j}. |

14. Consider the relation REFRIG (modelno, year, price, manufplant, color) which is abbreviated as REFRIG (m, y, p, mp, c) and the set of functional dependencies

 $F = \{m \rightarrow mp, \{m, y\} \rightarrow p, mp \rightarrow c\}$

- iv. Evaluate each of the following as candidate keys: $\{m\}$, $\{m, y\}$, $\{m, c\}$.
- v. Test if REFRIG is in 3NF? In BCNF?
- vi. Test if the decomposition $D = \{R1(m, y, p), R2(m, mp, c)\}$ is lossless.