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Interval Neutrosophic Sets

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ABSTRACT

A neutrosophic set is a part of neutrosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra. The neutrosophic set is a powerful general formal framework that has been recently proposed. However, the neutrosophic set needs to be specified from a technical point of view. Now we define the set-theoretic operators on an instance of a neutrosophic set, and call it an Interval Neutrosophic Set (INS). We prove various properties of INS, which are connected to operations and relations over INS. Finally, we introduce the convexity of interval neutrosophic sets.

KEY WORDS
Neutrosophic sets, interval neutrosophic sets, fuzzy sets, interval valued fuzzy sets, intuitionistic fuzzy sets, convexity

1 INTRODUCTION

The concept of fuzzy sets was introduced by Zadeh in 1965 [7]. Since then fuzzy sets and fuzzy logic have been applied in many real applications to handle uncertainty. The traditional fuzzy set uses one real number $\mu_A(x) \in [0,1]$ to represent the grade of membership of fuzzy set A defined on universe X. Sometimes $\mu_A(x)$ itself is uncertain and hard to be defined by a crisp value. So the concept of interval valued fuzzy sets was proposed [4] to capture the uncertainty of grade

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of membership. Interval valued fuzzy set uses an interval value $[\mu_A^L(x),\mu_A^U(x)]$ with $0 \le \mu_A^L(x) \le \mu_A^U(x) \le 1$ to represent the grade of membership of fuzzy set A. In some applications such as expert system, belief system and information fusion, we should consider not only the truth-membership supported by the evidence but also the false-membership against by the evidence. That is beyond the scope of fuzzy sets and interval valued fuzzy sets. In 1986, Atanassov introduced the intuitionistic fuzzy set [1] that is a generalization of fuzzy sets and provably equivalent to interval valued fuzzy set. The intuitionistic fuzzy sets consider both truth-membership and falsity-membership. Later on, intuitionistic fuzzy sets were extended to the interval valued intuitionistic fuzzy sets [2]. The interval valued intuitionistic fuzzy set uses a pair of intervals $[t^-,t^+]$, $0 \le t^- \le t^+ \le 1$ and $[f^-,f^+]$, $0 \le f^- \le f^+ \le 1$ with $t^+ + f^+ \le 1$ to describe the degree of true belief and false belief. Because of the restriction that $t^+ + f^+ \le 1$, intuitionistic fuzzy sets and interval valued intuitionistic fuzzy sets can only handle incomplete information not the indeterminate information and inconsistent information which exists commonly in real situations. For example, when we ask the opinion of an expert about certain statement, he or she may say that the possibility that the statement is true is between 0.5 and 0.7, and the statement is false is between 0.2 and 0.4, and the degree that he or she is not sure is between 0.1 and 0.3. Here is another example, suppose there are 10 voters during a voting process. In time t_1, three vote “yes”, two vote “no” and five are undecided, using neutrosophic notation, it can be expressed as x(0.3,0.5,0.2); in time t_2, three vote “yes”, two vote “no”, two give up and three are undecided, it then can be expressed as x(0.3,0.3,0.2). That is beyond the scope of the intuitionistic fuzzy set. So, the notion of neutrosophic set is more general and overcomes the aforementioned issues.

In neutrosophic set, indeterminacy is quantified explicitly and truth-membership, indeterminacy-membership and falsity-membership are independent. This assumption is very important in many applications such as information fusion in which we try to combine the data from different sensors. Neutrosophy was introduced by Smarandache in

1995. "It is a branch of philosophy which studies the origin, nature and scope of neutralities, as well as their interactions with different ideational spectra" [4]. Neutrosophic set is a powerful general formal framework which generalizes the concept of the classic set, fuzzy set [7], interval valued fuzzy set [5], intuitionistic fuzzy set [1], interval valued intuitionistic fuzzy set [2], paraconsistent set [4], etc. A neutrosophic set A defined on universe U. x = x(T,I,F) $\in$ A with T,I and F being the real standard or non-standard subsets of $]0^-,1^+[$, T is the degree of truth-membership of A, I is the degree of indeterminacy-membership of A and F is the degree of falsity-membership of A.

The neutrosophic set generalizes the above-mentioned sets from philosophical point of view. From scientific or engineering point of view, the neutrosophic set and set-theoretic operators need to be specified. Otherwise, it will be difficult to apply in the real applications. In this paper, we define the set-theoretic operators on an instance of neutrosophic set called interval neutrosophic set (INS). We call it as "interval" because it is subclass of neutrosophic set, that is we only consider the subunitary interval of [0,1].

An interval neutrosophic set A defined on universe X, x = x(T,I,F) $\in$ A with T,I and F being the subinterval of [0,1]. The interval neutrosophic set can represent uncertain, imprecise, incomplete and inconsistent information which exists in real world. The interval neutrosophic set generalizes the following sets:

1. the *classical set*, I = $\emptyset$, inf T = sup T = 0 or 1, inf F = sup F = 0 or 1 and sup T + sup F = 1.

2. the *fuzzy set*, I = $\emptyset$, inf T = sup T $\in$ [0,1], inf F = sup F $\in$ [0,1] and sup T + sup F = 1.

3. the *interval valued fuzzy set*, I = $\emptyset$, inf T, supT, inf F, sup F $\in$ [0,1], sup T + inf F = 1 and inf T + sup F = 1.

4. the *intuitionistic fuzzy set*, I = $\emptyset$, inf T = sup T $\in$ [0,1], inf F = sup F $\in$ [0,1] and sup T + sup F $\leq$ 1.

5. the *interval valued intuitionistic fuzzy set*, I = ∅, inf T, sup T, inf F, sup F ∈ [0,1] and sup T + sup F ≤ 1.

6. the *paraconsistent set*, I = ∅, inf T = sup T ∈ [0,1], inf F = sup F ∈ [0,1] and sup T + sup F > 1.

7. the interval valued paraconsistent set, I = ∅, inf T, sup T, inf F, sup F ∈ [0,1] and inf T + inf F > 1.

The relationship among the interval neutrosophic set and other sets is illustrated in Fig 1.

Note that → in Fig. 1 such as a → b means that b is a generalization of a.

We define the set-theoretic operators on the interval neutrosophic set (INS). Various properties of INS are proved, which are connected to the operations and relations over INS.

The rest of paper is organized as follows. Section 2 provides a brief overview of the neutrosophic set. Section 3 gives the definition of the interval neutrosophic set and set-theoretic operations. Section 4 presents some properties of set-theoretic operations. Section 5 defines the convexity of the interval neutrosophic sets and proves some properties of convexity. Section 6 concludes the paper.

2 NEUTROSOPHIC SET

This section gives a brief overview of concepts of the neutrosophic set defined in [4]. Here, we use different notations to express the same meaning. Let S_1 and S_2 be two real standard or non-standard subsets, then $S_1+S_2 = \{x|x = s_1+s_2, s_1 \in S_1 \text{ and } s_2 \in S_2\}$, $\{1^+\}+S_2 = \{x|x = 1^+ + s_2, s_2 \in S_2\}$. $S_1-S_2 = \{x|x = s_1 - s_2, s_1 \in S_1 \text{ and } s_2 \in S_2\}$, $\{1^+\}-S_2 = \{x|x = 1^+ - s_2, s_2 \in S_2\}$. $S_1*S_2 = \{x|x = s_1*s_2, s_1 \in S_1 \text{ and } s_2 \in S_2\}$.

Definition 1 (Neutrosophic Set) Let X be a space of points (objects), with a generic element in X denoted by x. A neutrosophic set A in X is characterized by a truth-membership function T_A, an indeterminacy-membership function I_A and a falsity-membership function F_A. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or non-standard subsets of $]0^-,1^+[$, that is

$$T_A : X \to]0^-,1^+[, \quad (1)$$

$$I_A : X \to]0^-,1^+[, \quad (2)$$

$$F_A : X \to]0^-,1^+[. \quad (3)$$

There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $0^- \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

Definition 2 (Complement) The complement of a neutrosophic set A is denoted by A' and is defined by

$$T_{A'}(x) = \{1^+\} - T_A(x), \quad (4)$$

$$I_{A'}(x) = \{1^+\} - I_A(x), \quad (5)$$

$$F_{A'}(x) = \{1^+\} - F_A(x), \quad (6)$$

for all x in X.

Definition 3 (Containment) A neutrosophic set A is contained in the other neutrosophic set B, $A \subseteq B$, if and only if

$$\inf T_A(x) \leq \inf T_B(x), \quad \sup T_A(x) \leq \sup T_B(x), \quad (7)$$

$$\inf F_A(x) \geq \inf F_B(x), \quad \sup F_A(x) \geq \sup F_B(x), \quad (8)$$

for all x in X

Definition 4 (Union) The union of two neutrosophic sets A and B is a neutrosophic set C, written as $C = A \cup B$, whose truth-membership, indeterminacy-membership and falsity-membership functions are related to those of A and B by

$$T_C(x) = T_A(x) + T_B(x) - T_A(x) * T_B(x), \quad (9).$$

$$I_C(x) = I_A(x) + I_B(x) - I_A(x) * I_B(x), \quad (10)$$

$$F_C(x) = F_A(x) + F_B(x) - F_A(x) * F_B(x), \quad (11)$$

for all x in X.

Definition 5 (Intersection) The intersection of two neutrosophic sets A and B is a neutrosophic set C, written as $C = A \cap B$, whose truth-membership, indeterminacy-membership and falsity-membership functions are related to those of A and B by

$$T_C(x) = T_A(x) * T_B(x), \quad (12)$$

$$I_C(x) = I_A(x) * I_B(x), \quad (13)$$

$$F_C(x) = F_A(x) * F_B(x), \quad (14)$$

for all x in X.

Definition 6 (Difference) The difference of two neutrosophic sets A and B is a

neutrosophic set C, written as C = A \ B, whose truth-membership, indeterminacy-membership and falsity-membership functions are related to those of A and B by

$$T_C(x) = T_A(x) - T_A(x) * T_B(x), \quad (15)$$

$$I_C(x) = I_A(x) - I_A(x) * I_B(x), \quad (16)$$

$$F_C(x) = F_A(x) - F_A(x) * F_B(x), \quad (17)$$

for all x in X.

3 INTERVAL NEUTROSOPHIC SET

In this section, we present the notion of *interval neutrosophic set* (INS). The interval neutrosophic set (INS) is an instance of the neutrosophic set which can be used in real scientific and engineering applications.

Definition 7 (Interval Neutrosophic Set) Let X be a space of points (objects), with a generic element in X denoted by x. An interval neutrosophic set (INS) A in X is characterized by truth-membership function T_A, indeterminacy-membership function I_A and falsity-membership function F_A. For each point x in X, $T_A(x), I_A(x), F_A(x) \subseteq [0,1]$.

An interval neutrosophic set (INS) A in R^1 is illustrated in Fig. 2.

When X is continuous, an INS A can be written as

$$A = \int_X < T(x), I(x), F(x) > / x, \; x \in X \quad (18)$$

When X is discrete, an INS A can be written as

$$A = \sum_{i=1}^{n} < T(xi), I(xi), F(xi) > / xi, \; x_i \in X \quad (19)$$

Consider parameters such as capability, trustworthiness and price of semantic Web services. These parameters are commonly used to define quality of service of semantic Web services. In this section, we will use the evaluation of quality of service of semantic Web services [6] as running example to illustrate every set-theoretic operation on the interval neutrosophic set.

Example 1 Assume that X = $\{x_1,x_2,x_3\}$. x_1 is capability, x_2 is trustworthiness and x_3 is price. The values of x_1, x_2 and x_3 are subset of [0,1]. They are obtained from the questionnaire of some domain experts, their option could be degree of good, degree of indeterminacy and degree of poor. A is an interval neutrosophic set of X defined by A = $<[0.2,0.4],[0.3,0.5],[0.3,0.5]>/x_1 + <[0.5,0.7],[0,0.2],[0.2,0.3]>/x_2 + <[0.6,0.8],[0.2,0.3],[0.2,0.3]>/x_3$. B is an interval neutrosophic set of X defined by B = $<[0.5,0.7],[0.1,0.3],[0.1,0.3]>/x_1 + <[0.2,0.3],[0.2,0.4],[0.5,0.8]>/x_2 + <[0.4,0.6],[0,0.1],[0.3,0.4]>/x_3$.

Definition 8 An interval neutrosophic set A is ***empty*** if and only if its inf $T_A(x)$ = sup $T_A(x) = 0$, inf $I_A(x)$ = sup $I_A(x) = 1$ and inf $F_A(x)$ = sup $T_A(x)$ = 0, for all x in X.

We now present the set-theoretic operators on the interval neutrosophic set.

Definition 9 Let A and B be two interval neutrosophic sets defined on X. **A(x)** $\leq$ **B(x)** if and only if

$$\inf T_A(x) \leq \inf T_B(x),\ \sup T_A(x) \leq \sup T_B(x), \quad (20)$$

$$\inf I_A(x) \leq \inf I_B(x),\ \sup I_A(x) \leq \sup I_B(x), \quad (21)$$

$$\inf F_A(x) \geq \inf F_B(x),\ \sup F_A(x) \geq \sup F_B(x). \quad (22)$$

Definition 10 (Containment) An interval neutrosophic set A is contained in the other interval neutrosophic set B, $A \subseteq B$, if and only if $A(x) \leq B(x)$, for all x in X.

Definition 11 Two interval neutrosophic sets A and B are ***equal***, written as A = B, if and only if $A \subseteq B$ and $B \subseteq A$.

Let N = $<[0,1]\times[0,1], [0,1]\times[0,1], [0,1]\times[0,1>$, $\underline{0} = <0,0,1>$, $\underline{1} = <1,1,0>$.

Definition 12 (Complement) Let C_N denote a neutrosophic complement of A. Then C_N is a function

$$C_N : N \rightarrow N$$

and C_N must satisfy at least the following three axiomatic requirements:

1) $C_N(\underline{0}) = \underline{1}$ and $C_N(\underline{1}) = \underline{0}$ (boundary conditions).
2) Let A and B be two interval neutrosophic sets defined on X, if $A(x) \leq B(x)$, then $C_N(A(x)) \geq C_N(B(x))$, for all x in X. (monotonicity)
3) Let A be an interval neutrosophic set defined on X, then $C_N(C_N(A(x))) = A(x)$, for all x in X. (involutivity).

There are many functions which satisfy the requirement to be the complement operator of interval neutrosophic sets. Here we give one example.

Definition 13 (Complement C_{N1}) The complement of an interval neutrosophic set A is denoted by A' and is defined by

$T_{A'}(x) = F_A(x),$ (23)

$\inf I_{A'}(x) = 1 - \sup I_{A'}(x),$ (24)

$\sup I_{A'}(x) = 1 - \inf I_A(x),$ (25)

$F_{A'}(x) = T_A(x),$ (26)

for all x in X.

Example 2 Let A be the interval neutrosophic set defined in Example 1. Then A' = <[0.3,0.5],[0.5,0.7],[0.2,0.4]>/x_1 + <[0.2,0.3],[0.8,1],[0.5,0.7]>/x_2 + <[0.2,0.3],[0.7,0.8],[0.6,0.8]>/x_3.

Definition 14 (N-norm) Let I_N denote a neutrosophic intersection of two interval neutrosophic sets A and B. Then I_N is a function

$I_N : N \times N \to N$

and I_N must satisfy at least the following four axiomatic requirements:

1) $I_N(A(x),\underline{1}) = A(x)$, for all x in X. (boundary condition).
2) $B(x) \le C(x)$ implies $I_N(A(x),B(x)) \le I_N(A(x),C(x))$, for all x in X. (monotonicity).
3) $I_N(A(x),B(x)) = I_N(B(x),A(x))$, for all x in X. (commutativity).
4) $I_N(A(x),I_N(B(x),C(x))) = I_N(I_N(A(x),B(x)),C(x))$, for all x in X. (associativity).

Here we give one example of intersection of two interval neutrosophic sets which

satisfies above N-norm axiomatic requirements. Other different definitions can be given for different applications.

Definition 15 (Intersection I_{N1}) The intersection of two interval neutrosophic sets A and B is an interval neutrosophic set C, written as $C = A \cap B$, whose truth-membership, indeterminacy-membership and falsity-membership functions are related to those of A and B by

$\inf T_C(x) = \min(\inf T_A(x), \inf T_B(x)),$ (27)

$\sup T_C(x) = \min(\sup T_A(x), \sup T_B(x))$, (28)

$\inf I_C(x) = \min(\inf I_A(x), \inf I_B(x))$, (29)

$\sup I_C(x) = \min(\sup I_A(x), \sup I_B(x))$, (30)

$\inf F_C(x) = \max(\inf F_A(x), \inf F_B(x))$, (31)

$\sup F_C(x) = \max(\sup F_A(x), \sup F_B(x))$, (32)

for all x in X.

Example 3 Let A and B be the interval neutrosophic sets defined in Example 1. Then, $A \cap B = <[0.2,0.4],[0.1,0.3],[0.3,0.5]>/x_1 + <[0.2,0.3],[0,0.2],[0.5,0.8]>/x_2 +$

$<[0.4,0.6],[0,0.1],[0.3,0.4]>/x_3$.

Theorem 1 $A \cap B$ is the largest interval neutrosophic set contained in both A and B.

Proof: It is easily verified from the definition of containment and intersection of interval neutrosophic sets. □

Definition 16 (N-conorm) Let U_N denote a neutrosophic union of two interval neutrosophic sets A and B. Then U_N is a function

$U_N : N \times N \to N$

and U_N must satisfy at least the following four axiomatic requirements:

1) $U_N(A(x),0) = A(x)$, for all x in X. (boundary condition).

2) $B(x) \leq C(x)$ implies $U_N(A(x),B(x)) \leq U_N(A(x),C(x))$, for all x in X. (monotonicity).

3) $U_N(A(x),B(x)) = U_N(B(x),A(x))$, for all x in X. (commutativity).

4) $U_N(A(x),U_N(B(x),C(x))) = U_N(U_N(A(x),B(x)),C(x))$, for all x in X. (associativity).

Here we give one example of union of two interval neutrosophic sets which satisfies above N-conorm axiomatic requirements. Other different definitions can be given for different applications.

Definition 17 (Union U_{N1}) The union of two interval neutrosophic sets A and B is an interval neutrosophic set C, written as $C = A \cup B$, whose truth-membership,

indeterminacy-membership and falsity-membership functions are related to those of A and B by

$\inf T_C(x) = \max(\inf T_A(x), \inf T_B(x)),$ (33)

$\sup T_C(x) = \max(\sup T_A(x), \sup T_B(x)),$ (34)

$\inf I_C(x) = \max(\inf I_A(x), \inf I_B(x)),$ (35)

$\sup I_C(x) = \max(\sup I_A(x), \sup I_B(x)),$ (36)

$\inf F_C(x) = \min(\inf F_A(x), \inf F_B(x)),$ (37)

$\sup F_C(x) = \min(\sup F_A(x), \sup F_B(x)),$ (38)

for all x in X.

Example 4 Let A and B be the interval neutrosophic sets defined in Example 1. Then, $A \cup B = <[0.5,0.7],[0.3,0.5],[0.1,0.3]>/x_1 + <[0.5,0.7],[0.2,0.4],[0.2,0.3]>/x_2 +$

$<[0.6,0.8],[0.2,0.3],[0.2,0.3]>/x_3$.

Theorem 2 $A \cup B$ is the smallest interval neutrosophic set containing both A and B.

Proof: It is straightforward from the definition of containment and union of interval neutrosophic sets. □

Theorem 3 Let P be the power set of all interval neutrosophic sets defined in the universe X. Then $<P; I_{N1}, U_{N1}>$ is a distributive lattice.

Proof: Let A, B, C be the arbitrary interval neutrosophic sets defined on X. It is easy to verify that $A \cap A = A$, $A \cup A = A$ (idempotency), $A \cap B = B \cap A$, $A \cup B = B \cup A$ (commutativity), $(A \cap B) \cap C = A \cap (B \cap C)$, $(A \cup B) \cup C = A \cup (B \cup C)$

(associativity), and $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$, $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ (distributivity). □

Theorem 4 $A \subseteq B \Leftrightarrow B' \subseteq A'$.

Proof: It is straightforward from the definition of containment and complement of

interval neutrosophic sets.

Definition 18 (Difference) The difference of two interval neutrosophic sets A and B is an interval neutrosophic set C, written as $C = A \setminus B$, whose truth-membership,

indeterminacy-membership and falsity-membership functions are related to those of A and B by

$\inf T_C(x) = \min(\inf T_A(x), \inf T_B(x))$, (39)

$\sup T_C(x) = \min(\sup T_A(x), \sup F_B(x))$, (40)

$\inf I_C(x) = \min(\inf I_A(x), 1 - \sup I_B(x))$, (41)

$\sup I_C(x) = \min(\sup I_A(x), 1 - \inf I_B(x))$, (42)

$\inf F_C(x) = \max(\inf F_A(x), \inf T_B(x))$, (43)

$\sup F_C(x) = \max(\sup F_A(x), \sup T_B(x))$, (44)

for all x in X.

Example 5 Let A and B be the interval neutrosophic sets defined in Example 1. Then, A \ B = <[0.1,0.3],[0.3,0.5],[0.5,0.7]>/x_1 + <[0.5,0.7],[0,0.2],[0.2,0.3]>/x_2 +

<[0.3,0.4],[0.1,0.3],[0.4,0.6]>/x_3.

Definition 19 (Addition) The addition of two interval neutrosophic sets A and B is an interval neutrosophic set C, written as C = A + B, whose truth-membership, indeterminacy-membership and falsity-membership functions are related to those of A and B by

$\inf T_C(x) = \min(\inf T_A(x) + \inf T_B(x), 1)$, (45)

$\sup T_C(x) = \min(\sup T_A(x) + \sup T_B(x), 1)$, (46)

$\inf I_C(x) = \min(\inf I_A(x) + \inf I_B(x), 1)$, (47)

$\sup I_C(x) = \min(\sup I_A(x) + \sup I_B(x), 1)$, (48)

$\inf F_C(x) = \min(\inf F_A(x) + \inf F_B(x), 1)$, (49)

$\sup F_C(x) = \min(\sup F_A(x) + \sup F_B(x), 1)$, (50)

for all x in X.

Example 6 Let A and B be the interval neutrosophic sets defined in Example 1. Then, A + B = <[0.7,1.0],[0.4,0.8],[0.4,0.8]>/x_1 + <[0.7,1.0],[0.2,0.6],[0.7,1.0]>/x_2 +

<[1.0,1.0],[0.2,0.4],[0.5,0.7]>/x_3.

Definition 20 (Scalar multiplication) The scalar multiplication of interval neutrosophic set A is an interval neutrosophic set B, written as B = a*A, whose truth-membership, indeterminacy-membership and falsity-membership functions are related to those of A by

$\inf T_B(x) = \min(\inf T_A(x) * a, 1)$, (51)

$\sup T_B(x) = \min(\sup T_A(x)*a,1),$ (52)

$\inf I_B(x) = \min(\inf I_A(x)*a,1),$ (53)

$\sup I_B(x) = \min(\sup I_A(x)*a,1),$ (54)

$\inf F_B(x) = \min(\inf F_A(x)*a,1),$ (55)

$\sup F_B(x) = \min(\sup F_A(x)*a,1),$ (56)

for all x in X.

Definition 21 (Scalar division) The scalar division of an interval neutrosophic set A is an interval neutrosophic set B, written as B = A / a, whose truth-membership, indeterminacy-membership and falsity-membership functions are related to those of A by

$\inf T_B(x) = \min(\inf T_A(x)/a,1),$ (57)

$\sup T_B(x) = \min(\sup T_A(x)/a,1),$ (58)

$\inf I_B(x) = \min(\inf I_A(x)/a,1),$ (59)

$\sup I_B(x) = \min(\sup I_A(x)/a,1),$ (60)

$\inf F_B(x) = \min(\inf F_A(x)/a,1),$ (61)

$\sup F_B(x) = \min(\sup F_A(x)/a,1),$ (62)

for all x in X, $a \in R^+$.

Now, we will define two operators: truth-favorite (Δ) and false-favorite(∇) to remove the indeterminacy in the interval neutrosophic sets and transform it into interval valued intuitionistic fuzzy sets or interval valued paraconsistent sets. These two operators are unique on interval neutrosophic sets.

Definition 22 (Truth-favorite) The truth-favorite of interval neutrosophic set A is an interval neutrosophic set Bm written as B = ΔA, whose truth-membership and falsity-membership functions are related to those of A by

$\inf T_B(x) = \min(\inf T_A(x)+\inf I_A(x),1),$ (63)

$\sup T_B(x) = \min(\sup T_A(x)+\sup I_A(x),1),$ (64)

$\inf I_B(x) = 0,$ (65)

$\sup I_B(x) = 0,$ (66)

$\inf F_B(x) = \inf F_A(x),$ (67)

$\sup F_B(x) = \sup F_A(x),$ (68)

for all x in X.

Example 7 Let A be the interval neutrosophic set defined in Example 1. Then, $\Delta A = <[0.5,0.9],[0,0],[0.3,0.5]>/x_1 + <[0.5,0.9],[0,0],[0.2,0.3]]>/x_2 + <[0.8,1.0],[0,0],[0.2,0.3]>/x_3$.

The purpose of truth-favorite operator is to evaluate the maximum of degree of truth-membership of interval neutrosophic set.

Definition 23 (False-favorite) The false-favorite of interval neutrosophic set A is an interval neutrosophic set B, written as $B = \nabla A$, whose truth-membership and falsity-membership functions are related to those of A by

$\inf T_B(x) = \inf T_A(x),$ (69)

$\sup T_B(x) = \sup T_A(x),$ (70)

$\inf I_B(x) = 0,$ (71)

$\sup I_B(x) = 0,$ (72)

$\inf F_B(x) = \min(\inf F_A(x) + \inf I_A(x),1),$ (73)

$\sup F_B(x) = \min(\sup F_A(x) + \sup I_A(x),1),$ (74)

for all x in X.

Example 8 Let A be the interval neutrosophic set defined in Example 1. Then, $\nabla A = <[0.2,0.4],[0,0],[0.6,1.0]>/x_1 + <[0.5,0.7],[0,0],[0.2,0.5]>/x_2 + <[0.6,0.8],[0,0],[0.4,0.6]>/x_3$.

The purpose of false-favorite operator is to evaluate the maximum of degree of falsity-membership of interval neutrosophic set.

Theorem 5 For every two interval neutrosophic sets A and B:

1. $\Delta(A \cup B) \subseteq \Delta A \cup \Delta B$
2. $\Delta A \cap \Delta B \subseteq \Delta(A \cap B)$
3. $\nabla A \cup \nabla B \subseteq \nabla(A \cup B)$
4. $\nabla(A \cap B) \subseteq \nabla A \cap \nabla B$

Proof: It is straightforward from the definition of truth-favorite and false-favorite.

4 PROPERTIES OF SET-THEORETIC OPERATORS

In this section, we will give some properties of set-theoretic operators defined on interval neutrosophic sets as in Section 3. The proof of these properties is left for the readers.

Property 1 (Commutativity) $A \cup B = B \cup A$, $A \cap B = B \cap A$, $A + B = B + A$.

Property 2 (Associativity) $A \cup (B \cup C) = (A \cup B) \cup C$, $A \cap (B \cap C) = (A \cap B) \cap C$, $A + (B + C) = (A + B) + C$.

Property3 (Distributivity) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

Property 5 $A \cap \Phi = \Phi$, $A \cup X = X$, where $\inf T_\Phi = \sup T_\Phi = \inf I_\Phi = \sup I_\Phi = 0$, $\inf F_\Phi = \sup F_\Phi = 1$ and $\inf T_X = \sup T_X = \inf I_X = \sup I_X = 1$, $\inf F_X = \sup F_X = 0$.

Property 6 $A \cup \Phi = A$, $A \cap X = A$, where $\inf T_\Phi = \sup T_\Phi = \inf I_\Phi = \sup I_\Phi = 0$, $\inf F_\Phi = \sup F_\Phi = 1$ and $\inf T_X = \sup T_X = \inf I_X = \sup I_X = 1$, $\inf F_X = \sup F_X = 0$.

Property 7 $\Delta(A + B) = \Delta A + \Delta B$, $\nabla(A + B) = \nabla A + \nabla B$.

Property 8 (Absorption) $A \cup (A \cap B) = A$, $A \cap (A \cup B) = A$.

Property 9 (De Morgan's Laws) $(A \cup B)' = A' \cap B'$, $(A \cap B)' = A' \cup B'$.

Property 10 (Involution) $(A')' = A$.

Here, we note that by the definition of complement, union and intersection of interval neutrosophic set, interval neutrosophic set satisfies the most properties of classic set, fuzzy set and intuitiionistic fuzzy set. Same as fuzzy set and intuitionistic fuzzy set, it does not satisfy the principle of middle exclude.

5 Convexity of Interval Neutrosophic Set

We assume that X is a real Euclidean space E^n for correctness.

Definition 24 (Convexity) An interval neutrosophic set A is convex if and only if

$$\inf T_A(\lambda x_1 + (1-\lambda)x_2) \geq \min(\inf T_A(x_1), \inf T_A(x_2)), \qquad (75)$$

$\sup T_A(\lambda x_1 + (1-\lambda)x_2) \geq \min(\sup T_A(x_1), \sup T_A(x_2))$, (76)

$\inf I_A(\lambda x_1 + (1-\lambda)x_2) \geq \min(\inf I_A(x_1), \inf I_A(x_2))$, (77)

$\sup I_A(\lambda x_1 + (1-\lambda)x_2) \geq \min(\sup I_A(x_1), \sup I_A(x_2))$, (78)

$\inf F_A(\lambda x_1 + (1-\lambda)x_2) \leq \max(\inf F_A(x_1), \inf F_A(x_2))$, (79)

$\sup F_A(\lambda x_1 + (1-\lambda)x_2) \leq \max(\sup F_A(x_1), \sup F_A(x_2))$, (80)

for all x_1 and x_2 in X and all $\lambda \in [0,1]$.

Fig. 2 is an illustration of a convex interval neutrosophic set.

Theorem 6 If A and B are convex, so is their intersection.

Proof: It is direct from the definition of convexity of interval neutrosophic sets.

Definition 25 (Strongly Convexity) An interval neutrosophic set A is strongly convex if for any two distinct points x_1 and x_2, and any λ in the open interval (0,1),

$\inf T_A(\lambda x_1 + (1-\lambda)x_2) > \min(\inf T_A(x_1), \inf T_A(x_2))$, (75)

$\sup T_A(\lambda x_1 + (1-\lambda)x_2) > \min(\sup T_A(x_1), \sup T_A(x_2))$, (76)

$\inf I_A(\lambda x_1 + (1-\lambda)x_2) > \min(\inf I_A(x_1), \inf I_A(x_2))$, (77)

$\sup I_A(\lambda x_1 + (1-\lambda)x_2) > \min(\sup I_A(x_1), \sup I_A(x_2))$, (78)

$\inf F_A(\lambda x_1 + (1-\lambda)x_2) < \max(\inf F_A(x_1), \inf F_A(x_2))$, (79)

$\sup F_A(\lambda x_1 + (1-\lambda)x_2) < \max(\sup F_A(x_1), \sup F_A(x_2))$, (80)

for all x_1 and x_2 in X and all $\lambda \in (0,1)$.

Theorem 7 If A and B are strongly convex, so is their intersection.

Proof: It is straightforward from the definition of strongly convexity of interval neutrosophic sets.

6 CONCLUSIONS AND FUTURE WORKS

In this paper, we have presented an instance of a neutrosophic set called an interval neutrosophic set. The interval neutrosophic set is an extension of classic sets, fuzzy sets, interval valued fuzzy sets, intuitionistic fuzzy sets, interval valued intuitionistic fuzzy sets, interval type-2 fuzzy sets [3] and paraconsistent sets. The notions of inclusion, union, intersection, complement and relation have been defined on the interval neutrosophic set. Various properties of set-theoretic operators have been proved. In the future, we will create a new logic system based on the interval neutrosophic set and apply the theory to solve practical applications in areas such as information fusion, data mining, bioinformatics, etc.

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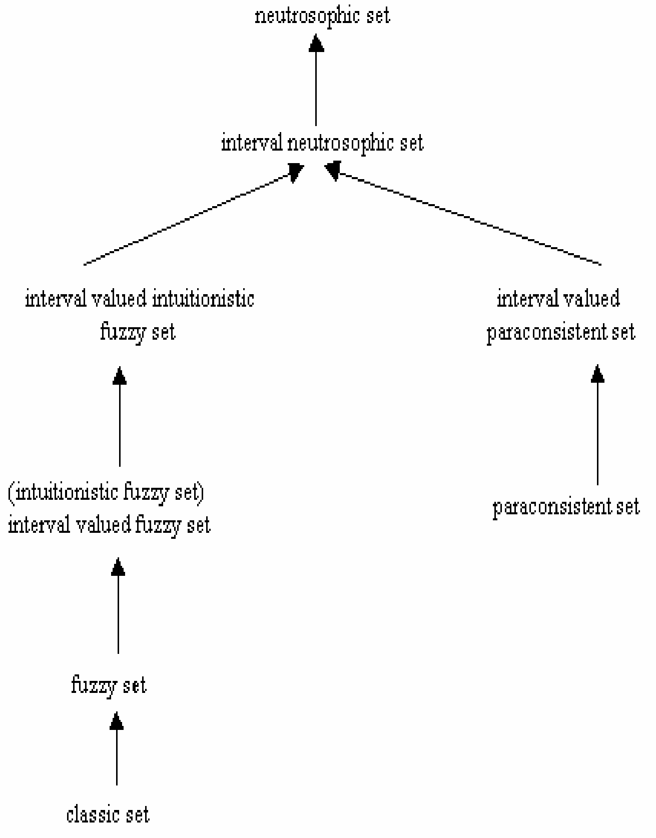

Figure 1
Relationship among the interval neutrosophic set and other sets

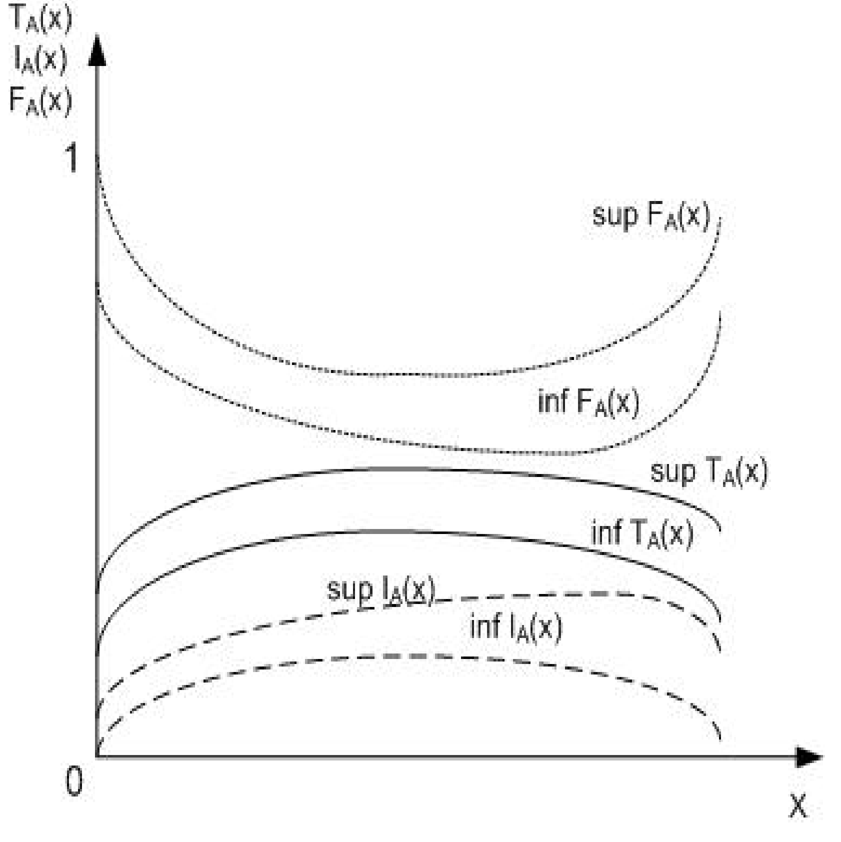

Figure 2
Illustration of interval neutrosophic set A in R^1