A Generalized SQL Query Construct for Paraconsistent Intuitionistic Fuzzy Databases

Haibin Wang
Department of Computer Science
Georgia State University
Atlanta, GA 30302, U.S.A.
hwang17@student.gsu.edu

Rajshekhar Sunderraman
Department of Computer Science
Georgia State University
Atlanta, GA 30302, U.S.A.
raj@cs.gsu.edu

Yan-Qing Zhang
Department of Computer Science
Georgia State University
Atlanta, GA 30302, U.S.A.
yzhang@cs.gsu.edu

Praveen Madiraju
Department of Computer Science
Georgia State University
Atlanta, GA 30302, U.S.A.
cscpnmx@cs.gsu.edu

***Abstract* – Recently, the paraconsistent intuitionistic fuzzy relational data model has been developed for representing and manipulating three kinds of uncertain information in databases: fuzzy, incomplete and inconsistent. The model is based on infinite-valued relations, called *paraconsistent intuitionistic fuzzy relations*. Here, we present an SQL-like SELECT statement construct for posing queries to paraconsistent intuitionistic fuzzy databases based on this model. The syntax of our generalized SELECT statement is similar to that of the usual SELECT statement of SQL, but its semantics is based on infinite-valued logic, making it an effective tool for querying paraconsistent intuitionistic fuzzy databases.**

I. Introduction

There are many real applications that need the capability of modeling and reasoning with fuzzy, incomplete and inconsistent information. Scientific data management is one such example. Consider an experimental scientist, e.g., a biologist, chemist, earth scientist, etc. Scientists typically conduct many experiments that produce raw data-sometimes vast amounts of data-that must be saved and analyzed. Data-generating experiments may be performed in a laboratory, in the field, with pencil-and-paper analysis, via computer simulation, or some combination of these methods. Regardless of the method, in many cases the data values to be stored may be fuzzy, incomplete, and inconsistent. Another example is sensor data management. Consider numerous sensors collecting data at regular intervals and transmitting their readings to a centralized system for processing. Often sensors may be unreliable: readings may be missed at some intervals, or transmitted values may be erroneous or imprecise.

Fuzzy set theory and fuzzy logic proposed by Zadeh [1] provides a requisite mathematical framework for dealing with fuzzy and uncertain information. Later on, the concept of interval-valued fuzzy sets was proposed to capture the fuzziness of grade of membership itself [2]. In 1986, Atanassov introduced the intuitionistic fuzzy set [3] which is a generalization of fuzzy set and provably equivalent to interval-valued fuzzy set. The intuitionistic fuzzy set considers both truth-membership t and falsity-membership f with $t, f \in [0, 1]$, and $0 \le t + f \le 1$. Because of this restriction, fuzzy sets, interval-valued fuzzy set and intuitionistic fuzzy sets cannot handle inconsistent information. Some authors [4-10] have studied relational databases in the light of fuzzy set theory with an object to accommodate a wider range of real world requirements and to provide closer man-machine interactions.

However, unlike fuzzy and incomplete information, inconsistent information has not enjoyed enough research attention. Generally, two basic approaches have been followed in solving the inconsistency problem in knowledge bases: belief revision and paraconsistent logics. The goal of the first approach is to make an inconsistent theory consistent, either by revising it or by representing it by a consistent semantics. On the other hand, the paraconsistent approach allows reasoning in the presence of inconsistency and contradictory information that can be derived or introduced without trivialization [11]. Bagai and Sunderraman [12] proposed a paraconsistent relational data model to deal with incomplete and inconsistent information. This data model is based on paraconsistent logics [13, 14].

Recently, Wang and Sunderraman [15] proposed a paraconsistent intuitionistic fuzzy relational data model. This data model is based on intuitionistic fuzzy logic and paraconsistent logics. The paraconsistent intuitionistic fuzzy relational data model can handle fuzzy, incomplete and inconsistent data.

Additionally, the model can readily underlie any database that needs to deal with fuzzy, incomplete and inconsistent information at the level of tuples. In this paper, we present an extension of the SQL SELECT statement for querying such databases. Our extension is based on the infinite-valued logic. While the syntax of this extended statement is similar to that of the ordinary SELECT statement, the new infinite-valued semantics that we propose is quite different. With our new extended semantics, the statement becomes an effective tool for querying paraconsistent intuitionistic fuzzy databases.

The remainder of this paper is organized as follows. Section 2 gives a quick overview of paraconsistent intuitionistic fuzzy relations on which the model of [15] is based. Section 3 presents the new infinite-valued SQL-like SELECT statement for querying paraconsistent intuitionistic fuzzy databases. The semantics of this SELECT statement is given in terms of extended algebraic operators that are defined in Section 4. Section 5 contains an example SELECT statement and a walk through the evaluation procedure for that query. Finally, Section 6 concludes the paper with some mention of related and future work directions.

II. Paraconsistent Intuitionistic Fuzzy Relations

In this section we briefly introduce paraconsistent intuitionistic fuzzy relations and their membership functions. Let a *relation scheme* (or just *scheme*) Σ be a finite set of *attribute names*, where for any attribute name $A \in \Sigma$, *dom(A)* is a non-empty *domain* of values for A. A *tuple* on Σ is any total map t: $\Sigma \rightarrow \cup_{A \in \Sigma} dom\ (A)$, such that $t(A) \in dom(A)$, for each $A \in \Sigma$. We let $\tau(\Sigma)$ denote the set of all tuples on Σ.

Definition 1 A *paraconsistent intuitionistic fuzzy relation* (or *p.i.f.r.* for short) R on a scheme Σ is any subset of $\tau(\Sigma)\times[0,1]\times[0,1]$.

For any $t \in \tau(\Sigma)$, we shall denote an element of R as $<t, R(t)^+, R(t)^->$, where $R(t)^+$ is the belief factor assigned to t by R and $R(t)^-$ is the doubt factor assigned to t by R. Note that since contradictory beliefs are possible, so $R(t)^+ + R(t)^-$ could be greater than 1. Furthermore, $R(t)^+ + R(t)^-$ could be less than 1, giving rise to incompleteness.

As an example, suppose in the e-shopping environment, there are two items I_1 and I_2, which are evaluated by customers for some categories of quality q_1, q_2 and q_3. Let the evaluation results be captured by the following *p.i.f.r.* EVAL on scheme $\{I, Q\}$:

EVAL

I_1	q_1	<0.9, 0.2>
I_1	q_2	<1.0, 0.0>
I_1	q_3	<0.1, 0.8>
I_2	q_1	<1.0, 1.0>
I_2	q_3	<0.8, 0.3>

The above p.i.f.r. contains the information that the confidence of item I_1 was evaluated "good" for category q_1 is 0.9 and the doubt is 0.2. The confidence of item I_1 was evaluated "good" for category q_2 is 1.0 and the doubt is 0.0. The confidence of item I_1 was evaluated "poor" for category q3 is 0.8 and the doubt is 0.1. Also, the confidence of item I_2 was evaluated "good" for category q_1 is 1.0 and the doubt is 1.0 (similarly, the confidence of item I_2 was evaluated "poor" for category q_1 is 1.0 and the doubt is 1.0). The confidence of I_2 was evaluated "good" for category q_3 is 0.8 and the doubt is 0.3. Note that the evaluation results of item I_2 for category q_2 is unknown. The above information contains results of fuzziness, incompleteness and inconsistency. Such information may be due to various reasons, such as evaluation not conducted, or evaluation results not yet available, the evaluation is not reliable, and different evaluation results for the same category producing different results, etc.

We define an infinite-valued membership function of a p.i.f.r., which maps tuples to the pair of values $<\alpha, \beta>$, with $0 \le \alpha+\beta \le 2$. We use the symbol **I** to denote the set of these values, i.e. $I = \{<\alpha, \beta>\}$. Now, for a p.i.f.r. $R = <t,R(t)^+,R(t)^->$ on scheme Σ, its membership function is an infinite-valued predicate $\Phi_R : \tau(\Sigma) \rightarrow \mathbf{I}$, given by

$$\Phi_R(t) = <R(t)^+, R(t)^->.$$

In [10], the authors have proposed a 4-valued characteristic function of paraconsistent relation, which maps tuples to one of the following values: ψ (for contradiction), t (for true), f (for false) and ζ (for unknown). It can be easily verified that when $R(t)^+ = R(t)^- = 1$, it corresponds to ψ; when $R(t)^+ = 1$, $R(t)^- = 0$, it corresponds to t; when $R(t)^+ = 0$, $R(t)^- = 1$, it corresponds to f; and when $R(t)^+ = R(t)^- = 0$, it corresponds to ζ. It is clear that our paraconsistent intuitionistic fuzzy relations are a generalization of paraconsistent relations which are a generalization of ordinary relations.

III. An infinite-valued select

The most popular construct for information retrieval from most commercial systems is the SQL's SELECT statement. While the statement has many options and extensions to its basic form, here we just present an infinite-valued generalization to the basic form, as generalizing the options then just becomes a trivial matter of detail. The basic form of the statement contains three clauses **select**, **from** and **where**, and has the following format:

select $A_1,A_2,\ldots,A_m$
from $R_1,R_2,\ldots,R_n$
where C

where

- $A_1,A_2,\ldots,A_m$ is a list of attribute names whose values are to be retrieved by the query,
- $R_1,R_2,\ldots,R_n$ is a list of relation names required to process the query, and
- C is a boolean expression that identifies the tuples to be retrieved by the query.

Without loss of generality, we assume that each attribute name occurs in exactly one relation, because if some attribute A_i occurs in more than one relation, we require, instead of simply the attribute A_i, a pair of the form $R_j.A_i$ qualifying that attribute.

The result of the SELECT statement is a relation with attributes $A_1,A_2\ldots,A_m$ chosen from the attributes of $R_1 \times R_2 \times \ldots \times R_n$, for tuples that satisfy the boolean condition C, i.e.

$$\pi_{A1,A2,\ldots,Am}(\sigma_C(R_1 \times R_2 \times \ldots \times R_n)),$$

where π, σ and $\times$ are the projection, selection and product operations, respectively, on ordinary relations.

We retain the above syntax in the generalized SELECT statement for paraconsistent intuitionistic fuzzy relations. However, the relation names $R_1,R_2,\ldots,R_n$ now represent some paraconsistent intuitionistic fuzzy relations and C is some infinite-valued condition. The result of the generalized SELECT statement is then the value of the algebraic expression:

$$\pi^p_{A1,A2,\ldots,Am}(\sigma^p_C(R_1 \times^p R_2 \times^p \ldots \times^p R_n)),$$

where π^p, σ^p, $\times^p$ are, respectively, the projection, selection and product operations on paraconsistent intuitionistic fuzzy relations constructed in the next section. Furthermore, the result of the generalized SELECT statement is also a paraconsistent intuitionistic fuzzy relation.

IV. ALBEBRAIC OPERATORS

In Section II, we observed that paraconsistent intuitionistic fuzzy relations are a generalization of paraconsistent relations which are a generalization of ordinary relations, in that for each paraconsistent relation there is a paraconsistent intuitionistic fuzzy relation with the same information content, but not *vice versa*. To complete the semantics of the infinite-valued SELECT statement, we now generalize the required algebraic operators on paraconsistent relations (product, selection and projection) for paraconsistent intuitionistic fuzzy relations. To reflect such a generalization, a superscript "p" is placed aside an ordinary relation operator to obtain the corresponding paraconsistent intuitionistic fuzzy relation operator.

We first introduce some notation. If $\sum$ and Δ are relation schemes such that $\sum \subseteq \Delta$, then for any tuple $t \in \tau(\sum)$, we let t^{Δ} denote the set

$$\{t' \in \tau(\Delta) \mid t'(A) = t(A), \text{ for all } A \in \sum\}$$

of all extensions of t. In other words, t^{Δ} is the set of all tuples on Δ that agree with t for all attributes of $\sum$. Clearly, t^{Δ} will be infinite iff the domain of at least one attribute in Δ - $\sum$ is infinite. We extend this notion for any $T \subseteq \tau(\sum)$ by defining $T^{\Delta} = \cup_{t\in T}\, t^{\Delta}$.

A. Projection (π^p) and product ($\times^p$)

We now define the projection operator on paraconsistent intuitionistic fuzzy relations.

Definition 2 Let R be a paraconsistent intuitionistic fuzzy relation on scheme $\sum$, and $\Delta \subseteq \sum$ be any scheme. Then, the **projection** of R onto Δ, denoted $\pi^{p}_{\Delta}(R)$, is a paraconsistent intuitionistic fuzzy relation on scheme Δ, given by

$$(\pi^{p}_{\Delta}(R))(t) = <\max\{R(u)^{+} \mid u \in t^{\Sigma}\},\min\{R(u)^{-} \mid u \in t^{\Sigma}\}>.$$

The belief factor of a tuple in the projection is the maximum of the belief factors of all of the tuple's extensions onto the scheme of the input paraconsistent intuitionistic fuzzy relation. Moreover, the doubt factor of a tuple in the projection is the minimum of the doubt factors of all of the tuple's extensions onto the scheme of the input paraconsistent intuitionistic fuzzy relation.

The product operator on paraconsistent intuitionistic fuzzy relations is defined in terms of natural join among them.

Definition 3 Let R and S be paraconsistent intuitionistic fuzzy relations on schemes $\sum$ and Δ, respectively. Then, the **natural join** (further for short called **join**) of R and S, denoted $R \infty^{p} S$, is a paraconsistent intuitionistic fuzzy relation on scheme $\sum \cup \Delta$, given by

$$R \infty^{p} S(t) = <\min\{R(\pi_{\Sigma}(t))+,S(\pi_{\Delta}(t))+\},\max\{ R(\pi_{\Sigma}(t))+,S(\pi_{\Delta}(t))+\}>,$$

Where π is the usual projection of a tuple.

The product of paraconsistent intuitionistic fuzzy relations R and S is essentially a join after renaming their attributes to make their schemes disjoint. Let $\rho(R)$ be the paraconsistent intuitionistic fuzzy relation with the same tuples in R, but with attribute names of the form "*R.A*" for each attribute name A of R.

Definition 4 For any paraconsistent intuitionistic fuzzy relations R and S, $R \times p\, S = \rho(R) \infty^{p}\rho(S)$.

B. Infinite-valued conditions

In the generalized SELECT statement, we let the condition occurring in the **where** clause be infinite-valued. The infinite values, except <1, 0> and <0, 1>, arise essentially due to any nested subqueries. For any arithmetic expressions E_1 and E_2, comparisons such as $E_1 \le E_2$ are simply 2-valued conditions (<1,0> or <0,1>).

Let $\Im$ be a subquery of the form

(**select** … **from** … **where** …)

occurring in the **where** clause of a SELECT statement. And let R be the paraconsistent intuitionistic fuzzy relation on scheme $\sum$ that the subquery $\Im$ evaluates to. Then, conditions involving the subquery $\Im$ evaluate as follows.

1) The condition

exists $\Im$

evaluates to $<\alpha, \beta>$,

$\alpha = \max\{a\}$, $a = R(t)^{+}$, for all $t \in \tau(\sum)$,

$\beta = \min\{b\}$, $b = R(t)^{-}$, if $R(t)^{+} + R(t)^{-} \le 1$, $b = 1 - R(t)^{+}$, if $R(t)^{+} + R(t)^{-} > 1$, for all $t \in \tau(\sum)$.

2) For any tuple $t \in \tau(\sum)$, the condition

t **in** $\Im$

evaluates to $\Phi_R(t)$.

3) If $\sum$ contains exactly one attribute, then for any (scalar value) $t \in \tau(\sum)$, the condition

t >**any** $\Im$

evaluates to $<\alpha, \beta>$,

$\alpha = \max\{a\}$, $a = R(k)^{+}$, if $t > k$, for some $k \in$ R,

($\beta = \min\{b\}$, $b = R(k)^{-}$, if $R(k)^{+}+R(k)^{-} \le 1$, $b = 1-R(k)^{+}$, if $R(k)^{+}+R(k)^{-} > 1$), if $t > k$, for some $k \in$ R;

$\alpha = 0$, $\beta = 1$, otherwise.

An infinite-valued semantics for other operators, such as ≥**any**, =**any**, can be defined similarly. Note that conditions involving such operators never evaluate to the value $<\alpha, \beta>$, such that $\alpha + \beta > 1$.

4) If $\sum$ contains exactly one attribute, then for any (scalar value) $t \in \tau(\sum)$, the condition

t >**all** $\Im$

evaluates to $<\alpha, \beta>$,

($\alpha = \min\{a\}$, $a = R(k)^{-}$, if $R(k)^{+}+R(k)^{-} \le 1$, $a = 1-R(k)^{+}$, if $R(k)^{+}+R(k)^{-} > 1$), if $t \le k$, for some $k \in$ R,

$\beta = \max\{b\}$, $b = R(k)^{+}$, if $t \le k$, for some $k \in$ R;

$\alpha = 1$, $\beta = 0$, otherwise.

An infinite-valued semantics for other operators, such as ≥**all**, =**all**, can be defined similarly. Note that conditions involving such operators never evaluate to the value $<\alpha, \beta>$, such that $\alpha + \beta > 1$.

We complete our infinite-valued semantics for conditions by defining the **not**, **and** and **or** operators on them. Let C and D be any conditions, and value of $C = <t_c, f_c>$ and value of $D = <t_d, f_d>$. Then, the value of the condition **not** C is given by

not $C = <f_c, t_c>$

while the value of the condition C **and** D is given by

$$C \textbf{ and } D = \langle \min\{t_c, t_d\}, \max\{f_c, f_d\}\rangle$$

and that of the condition C **or** D is given by

$$C \textbf{ or } D = \langle \max\{t_c, t_d\}, \min\{f_c, f_d\}\rangle$$

The duality of **and** and **or** is evident from their formulas. It is interesting to note the following algebraic laws exhibited by the above infinite-valued operators:

1. Double Complementation Law:
   not (**not** C) = C
2. Identity and Idempotence Laws:
   C **and** <1, 0> = C **and** C = C
   C **or** <0, 1> = C **or** C = C
3. Commutativity Laws:
   C **and** D = D **and** C
   C **or** D = D **or** C
4. Associativity Laws:
   C **and** (D **and** E) = (C **and** D) **and** E
   C **or** (D **or** E) = (C **or** D) **or** E
5. Distributivity Laws:
   C **and** (D **or** E) = (C **and** D) **or** (C **and** E)
   C **or** (D **and** E) = (C **or** D) **and** (C **or** E)
6. De Morgan Laws:
   not (C **and** D) = (**not** C) **or** (**not** D)
   not (C **or** D) = (**not** C) **and** (**not** D)

C. Selection (σ^p)

We are now ready to define the selection operator on paraconsistent intuitionistic fuzzy relations.

Definition 5 Let R be a paraconsistent intuitionistic fuzzy relation on scheme Σ, and C be an infinite-valued condition on tuples of Σ denoted $\langle t_C(t), f_C(t)\rangle$. Then, the selection of R by C, denoted $\sigma^p{}_C(R)$, is a paraconsistent intuitionistic fuzzy relation on scheme Σ, given by

$$(\sigma^p{}_C(R))(t) = \langle \min\{R(t)^+, t_C(t)\}, \max\{R(t)^-, f_C(t)\}\rangle.$$

The above definition is similar to that of the **and** operator given earlier.

D. Union

Since performing a simple union is impossible within a SELECT statement, SQL provides a **union** operator among subqueries to achieve this. We end this section with an infinite-valued semantics of **union**.

Let $\Im_1$ and $\Im_2$ be subqueries that evaluate, respectively, to paraconsistent intuitionistic fuzzy relations R_1 and R_2 on scheme Σ. Then, the subquery

$$\Im_1 \textbf{ union } \Im_2$$

evaluates to the paraconsistent intuitionistic fuzzy relation R on scheme Σ given by

$$R(t) = \langle \max\{R_1(t)^+, R_2(t)^+\}, \min\{R_1(t)^-, R_2(t)^-\}\rangle$$

An intuitive appreciation of the union operator can be obtained as follows: Given a tuple t, since we believed that it is present in the relation R_1 with confidence $R_1(t)^+$ and that it is present in the relation R_2 with confidence $R_2(t)^+$, we can now believe that the tuple t is present in the "either-R_1-or-R_2" relation with confidence which is equal to the larger of $R_1(t)^+$ and $R2(t)^+$. Using the same logic, we can now believe in the absence of the tuple t from the "either-R_1-or-R_2" relation with confidence which is equal to the smaller (because t must be absent from both R_1 and R_2 for it to be absent from the union) of $R_1(t)^-$ and $R_2(t)^-$.

V. An Example

Let us now consider an example illustrating some infinite-valued computations. We reproduce here the paraconsistent intuitionistic fuzzy relation EVAL on scheme $\{I,Q\}$ of the item-quality example of Section II:

EVAL

I_1	q_1	<0.9, 0.2>
I_1	q_2	<1.0, 0.0>
I_1	q_3	<0.1, 0.8>
I_2	q_1	<1.0, 1.0>
I_2	q_3	<0.8, 0.3>

Consider the query:

What items showed contradictory evaluation of some category of quality?

A SELECT statement for this query is:

select I
from $EVAL$
where **not** ((I, Q) **in** $EVAL$)

One possible evaluation method for the above query in ordinary 2-valued SQL is to produce the I attribute of those rows of $EVAL$ that satisfy the **where** condition. Since the **where** condition in the above case is exactly that row *not* be in $EVAL$, in 2-valued logic the above query will produce an empty answer.

In infinite-valued logic, however, the where condition needs to be evaluated, to one of infinite possible values, for every possible row with attributes Σ = (I, Q). The result is then combined with $EVAL$ according to the semantics of σ^p, on which π^p is performed to produce the resulting paraconsistent intuitionistic fuzzy relation.

Therefore, for each of the 6 rows in $\tau(\Sigma)$, we first evaluate the **where** condition C:

(I, Q)	C = **not** ((I,Q) **in** $EVAL$)
(I_1,q_1)	<0.2, 0.9>
(I_1,q_2)	<0.0, 1.0>
(I_1,q_3)	<0.8, 0.1>
(I, Q)	C = **not** ((I,Q) **in** $EVAL$)
(I_2,q_1)	<1.0, 1.0>
(I_2,q_2)	<0.0, 0.0>
(I_2,q_3)	<0.3, 0.8>

Now, $\sigma^p{}_C(EVAL)$ according to the definition of σ^p evaluates to the paraconsistent intuitionistic fuzzy relation:

$\sigma^p_C(EVAL)$

I_1	q_1	<0.2, 0.9>
I_1	q_2	<0.0, 1.0>
I_1	q_3	<0.1, 0.8>
I_2	q_1	<1.0, 1.0>
I_2	q_3	<0.3, 0.8>

Finally, π^p of the above is the paraconsistent intuitionistic fuzzy relation:

$\pi^p_I(\sigma^p_C(EVAL))$

I_1	<0.2, 0.8>
I_2	<1.0, 0.0>

which is the result of the SELECT statement. The result states that I_1 showed contradictory evaluation result for some category (actually q_1) with belief confidence 0.2 and doubt confidence 0.8, and I2 showed contradictory evaluation result for some category (actually q_1) with belief confidence 1.0 and doubt confidence 0.0.

VI. CONCLUSIONS

We have presented an infinite-valued semantics for the SQL SELECT statement for querying paraconsistent intuitionistic fuzzy databases. Our semantics is based on paraconsistent intutitionistic fuzzy relations of Wang and Sunderraman [15] in which fuzzy, incomplete and inconsistent information about tuples

We have given the extended semantics for the SELECT statement based on the extended algebraic operators (projection, selection and product) on paraconsistent intuitionistic fuzzy relations. These operators are true generalization of the corresponding operators on fuzzy relations and paraconsistent relations.

We also extended SQL's Boolean conditions to infinite-valued logic. The infinite values arise essentially due to sub queries in the SELECT statement's **where** clause, involving predicates such as **exists**, **in**, >**any** and >**all**. The extended infinite-valued semantics of the **not**, **and** and **or** operators on conditions were shown to possess elegant algebraic laws.

Some future directions in which we plan to extend this work are to develop syntax and semantics for the update statement on paraconsistent intuitionistic fuzzy databases and develop tuple-relational and domain-relational calculus for the paraconsistent intuitionistic fuzzy databases.

REFERENCES

[1] L.A. Zadeh, "Fuzzy sets," *Inf. Control* vol. 8, pp. 338-353, 1965.

[2] I. Turksen, "Interval valued fuzzy sets based on normal form," *Fuzzy Sets and Systems* vol 20, pp. 191-210, 1986

[3] K. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 20, pp. 271-350, 1986

[4] M. Anvari and G.F. Rose, "Fuzzy relational databases," Proceedings of the 1st International Conference on Fuzzy Information Processing, CRC Press, 1984

[5] J.F. Baldwin, "A fuzzy relational inference language for expert systems," Proceedings of the 13th IEEE International Symposium on Multivalued Logic. pp. 416-423, 1983

[6] B.P. Buckles and F. E. Petry, "A fuzzy representation for relational databases," *Fuzzy Sets and Systems*, vol. 7, pp.213-226, 1982

[7] S.K. Chang and J.S. Ke, "Database skeleton and its application to fuzzy query translation," *IEEE Transactions on Software Engineering*, vol. 4, pp. 31-43, 1978

[8] J. Kacprzyk and A. Ziolkowski, "Database queries with fuzzy linguistic quantifiers*," IEEE Transactions on SMC*, vol. 3, pp. 474-479, 1986

[9] H. Prade, "Lipski's approach to incomplete information databases restated and generalized in the setting of zadeh's possibility theory," *Information System* vol. 1 pp. 27-42, 1984

[10] K.V.S.V.N. Raju and A.K. Majumdar, "Fuzzy functional dependencies and lossless join decomposition of fuzzy relational databases systems," *ACM Transactions on Database Systems*, vol. 2, pp. 129-166, 1988

[11] S. de Amo, W. Carnielli and J. Marcos, "A logical framework for integrating inconsistent information in multiple databases," Proceedings of PoIKS' 02, LNCS 2284, pp. 67-84, 2002

[12] R. Bagai and R. Sunderraman, "A paraconsistent relational data model," *International Journal of Computer Mathematics*, vol. 55, August 1995

[13] N.D. Belnap, "A useful four-valued logic," Modern Uses of Many-valued Logic, Reidel, Dordrecht, pp. 8-37, 1977

[14] N.C.A.D. Costa, "On the theory of inconsistent formal systems," *Notre Dame Journal of Formal Logic* vol. 15, pp. 621-630, 1977

[15] H. Wang and R. Sunderraman, "A data model based on paraconsistent intuitionistic fuzzy relations," Proceedings of 15th International Symposium on Methodology for Intelligent System, 2005, in press.