

A Data Model Based on Paraconsistent Intuitionistic Fuzzy Relations

Haibin Wang and Rajshekhar Sunderraman

Department of Computer Science,
Georgia State University,
Atlanta, GA 30302, USA
hwang17@student.gsu.edu, raj@cs.gsu.edu

Abstract. Paraconsistent intuitionistic fuzzy set is an extension of intuitionistic fuzzy set or interval-valued fuzzy set. It relaxes the requirement that $t + f \leq 1$, where t is grade of truth-membership and f is grade of false-membership. In paraconsistent intuitionistic fuzzy set, $t, f \in [0, 1], 0 \leq t + f \leq 2$. In this paper, we present a generalization of the relational model of data based on paraconsistent intuitionistic fuzzy set. Our data model is capable of manipulating incomplete as well as inconsistent information. Associated with each relation there are two membership functions which keep track of the extent to which we believe the tuple is in the relation and the extent to which we believe that it is not in the relation. In order to handle inconsistent situations, we propose an operator, called “split”, to transform inconsistent paraconsistent intuitionistic fuzzy relations into pseudo-consistent paraconsistent intuitionistic fuzzy relations. We may then manipulate these pseudo-consistent paraconsistent intuitionistic fuzzy relations by performing set-theoretic and relation-theoretic operations on them. Finally, we can use another operator, called “combine”, to transform the results back to paraconsistent intuitionistic fuzzy relations. For this model, we define algebraic operators that are generalization of the usual operators such as union, selection, join on fuzzy relations. Our data model can underlie any database management system that deals with incomplete or inconsistent information.

1 Introduction

The relational model was proposed by Ted Codd’s in his pioneering paper [1]. This data model usually takes care of only well-defined and unambiguous data. However, imperfect information is prevalent in many situations and approaches to deal with such imperfect data need to be devised.

In order to represent and manipulate various forms of incomplete information in relational databases, several extensions of the classical relational model have been proposed [2, 3, 4]. In some of these extensions, a variety of “null values” have been introduced to model unknown or not-applicable data values. Attempts have also been made to generalize operators of relational algebra to manipulate such extended data models [3, 5]. Fuzzy set theory and fuzzy logic proposed by Zadeh

[6] provides a requisite mathematical framework for dealing with incomplete and imprecise information. Later on, the concept of interval-valued fuzzy sets was proposed to capture the fuzziness of grade of membership itself [7]. In 1986, Atanassov introduced the intuitionistic fuzzy set [8] which is a generalization of fuzzy set and provably equivalent to interval-valued fuzzy set. The intuitionistic fuzzy set considers both truth-membership t and false-membership f with $t, f \in [0, 1]$ and $t + f \leq 1$. Because of this restriction, fuzzy sets, interval-valued fuzzy sets and intuitionistic fuzzy sets cannot handle inconsistent information. Some authors [9, 10, 11, 12, 13, 14] have studied relational databases in the light of fuzzy set theory with an objective to accommodate a wider range of real-world requirements and to provide closer man-machine interactions.

However, unlike incomplete, imprecise, and uncertain information, inconsistent information has not enjoyed enough research attention. In fact, inconsistent information exists in a lot of applications. For example, in data warehousing application, inconsistency will appear when trying to integrate data from many different sources. Another example is that of expert systems, where the knowledge base itself may contain facts which are inconsistent with each other. Generally, two basic approaches have been followed in solving the inconsistency problem in knowledge bases: belief revision and paraconsistent logics. The goal of the first approach is to make an inconsistent theory consistent, either by revising it or by representing it by a consistent semantics. On the other hand, the paraconsistent approach allows reasoning in the presence of inconsistency and contradictory information can be derived or introduced without trivialization [15]. Bagai and Sunderraman [16] proposed a paraconsistent relational data model to deal with incomplete and inconsistent information. This data model is based on paraconsistent logics.

In this paper, we present a new relational data model - paraconsistent intuitionistic fuzzy relational data model (PIFRDM). Our model is based on the intuitionistic fuzzy set theory [8] and is capable of manipulating incomplete as well as inconsistent information. Let X be the universe of discourse. We use both grade of truth membership α and grade of false membership β to denote the status of a tuple of a certain relation with $\alpha(x), \beta(x) \in [0, 1]$ and $\alpha(x) + \beta(x) \leq 2$ for all $x \in X$. PIFRDM is the generalization of fuzzy relational data model (FRDM). That is, when $\alpha(x) + \beta(x) = 1$ for all $x \in X$, paraconsistent intuitionistic fuzzy relation is the ordinary fuzzy relation. This generalization is distinct from paraconsistent relational data model (PRDM), in fact it can be easily shown that PRDM is a special case of PIFRDM. That is, when $\alpha(x), \beta(x) = 0$ or 1 for all $x \in X$, paraconsistent intuitionistic fuzzy relation is just paraconsistent relation.

We introduce paraconsistent intuitionistic fuzzy relations, which are the fundamental mathematical structures underlying our model. These structures are strictly more general than classical fuzzy relations, in that for any fuzzy relation there is a paraconsistent intuitionistic fuzzy relation with the same information content, but not *vice versa*. The claim is also true for the relationship between paraconsistent intuitionistic fuzzy relations and paraconsistent relations. We define algebraic operators over paraconsistent intuitionistic fuzzy relations that extend the standard operators, such as selection, join, union, over fuzzy relations.

2 Paraconsistent Intuitionistic Fuzzy Relations

In this section, we generalize fuzzy relations and paraconsistent relations in such a manner that we are now able to assign a measure of belief and a measure of doubt to each tuple. We shall refer to these generalized relations as *paraconsistent intuitionistic fuzzy relations*. So, a tuple in a paraconsistent intuitionistic fuzzy relation is assigned a measure $\langle \alpha, \beta \rangle$, $0 \leq \alpha, \beta \leq 1$. α will be referred to as the *belief* factor and β will be referred to as the *doubt* factor. The interpretation of this measure is that we believe with confidence α and doubt with confidence β that the tuple is in the relation. The belief and doubt confidence factors for a tuple need not add to exactly 1. This allows for incompleteness and inconsistencies to be represented. If the belief and doubt factors add up to less than 1, we have incomplete information regarding the tuple's status in the relation and if the belief and doubt factors add up to more than 1, we have inconsistent information regarding the tuple's status in the relation.

We now formalize the notion of a paraconsistent intuitionistic fuzzy relation.

Let a *relation scheme* (or just *scheme*) Σ be a finite set of *attribute names*, where for any attribute $A \in \Sigma$, $dom(A)$ is a non-empty *domain* of values for A . A *tuple* on Σ is any map $t : \Sigma \rightarrow \cup_{A \in \Sigma} dom(A)$, such that $t(A) \in dom(A)$, for each $A \in \Sigma$. Let $\tau(\Sigma)$ denote the set of all tuples on Σ .

Definition 1. A **paraconsistent intuitionistic fuzzy relation** R on scheme Σ is any subset of $\tau(\Sigma) \times [0, 1] \times [0, 1]$. For any $t \in \tau(\Sigma)$, we shall denote an element of R as $\langle t, R(t)^+, R(t)^- \rangle$, where $R(t)^+$ is the belief factor assigned to t by R and $R(t)^-$ is the doubt factor assigned to t by R . Let $\mathcal{V}(\Sigma)$ be the set of all paraconsistent intuitionistic fuzzy relations on Σ .

Definition 2. A paraconsistent intuitionistic fuzzy relation R on scheme Σ is **consistent** if $R(t)^+ + R(t)^- \leq 1$, for all $t \in \tau(\Sigma)$. Let $\mathcal{C}(\Sigma)$ be the set of all consistent paraconsistent intuitionistic fuzzy relations on Σ . R is said to be **complete** if $R(t)^+ + R(t)^- \geq 1$, for all $t \in \tau(\Sigma)$. If R is both consistent and complete, i.e. $R(t)^+ + R(t)^- = 1$, for all $t \in \tau(\Sigma)$, then it is a **total paraconsistent intuitionistic fuzzy relation**, and let $\mathcal{T}(\Sigma)$ be the set of all total paraconsistent intuitionistic fuzzy relations on Σ .

Definition 3. R is said to be **pseudo-consistent** if $\max\{b_i | (\exists t \in \tau(\Sigma))(\exists d_i)((t, b_i, d_i) \in R)\} + \max\{d_i | (\exists t \in \tau(\Sigma))(\exists b_i)((t, b_i, d_i) \in R)\} > 1$, where for these (t, b_i, d_i) , $b_i + d_i = 1$. Let $\mathcal{P}(\Sigma)$ be the set of all pseudo-consistent paraconsistent intuitionistic fuzzy relations on Σ .

It should be observed that total paraconsistent intuitionistic fuzzy relations are essentially fuzzy relations where the uncertainty in the grade of membership is eliminated. Let $\mathcal{F}(\Sigma)$ be the set of all fuzzy relations over scheme Σ . We make this relationship explicit by defining an one-one correspondence $\lambda_\Sigma : \mathcal{T}(\Sigma) \rightarrow \mathcal{F}(\Sigma)$, given by $\lambda_\Sigma(R)(t) = R(t)^+$, for all $t \in \tau(\Sigma)$. This correspondence is used frequently in the following discussion.

Operator Generalisations

It is easily seen that paraconsistent intuitionistic fuzzy relations are a generalisation of fuzzy relations, in that for each fuzzy relation there is a paraconsistent intuitionistic fuzzy relation with the same information content, but not *vice versa*. It is thus natural to think of generalising the operations on fuzzy relations, such as union, join, projection etc., to paraconsistent intuitionistic fuzzy relations. However, any such generalisation should be intuitive with respect to the belief system model of paraconsistent intuitionistic fuzzy relations. We now construct a framework for operators on both kinds of relations and introduce two different notions of the generalisation relationship among their operators.

An n -ary operator on fuzzy relations with signature $\langle \Sigma_1, \dots, \Sigma_{n+1} \rangle$ is a function $\Theta : \mathcal{F}(\Sigma_1) \times \dots \times \mathcal{F}(\Sigma_n) \rightarrow \mathcal{F}(\Sigma_{n+1})$, where $\Sigma_1, \dots, \Sigma_{n+1}$ are any schemes. Similarly, an n -ary operator on paraconsistent intuitionistic fuzzy relations with signature $\langle \Sigma_1, \dots, \Sigma_{n+1} \rangle$ is a function $\Psi : \mathcal{V}(\Sigma_1) \times \dots \times \mathcal{V}(\Sigma_n) \rightarrow \mathcal{V}(\Sigma_{n+1})$.

Definition 4. An operator Ψ on paraconsistent intuitionistic fuzzy relations with signature $\langle \Sigma_1, \dots, \Sigma_{n+1} \rangle$ is **totality preserving** if for any total paraconsistent intuitionistic fuzzy relations R_1, \dots, R_n on schemes $\Sigma_1, \dots, \Sigma_n$, respectively, $\Psi(R_1, \dots, R_n)$ is also total.

Definition 5. A totality preserving operator Ψ on paraconsistent intuitionistic fuzzy relations with signature $\langle \Sigma_1, \dots, \Sigma_{n+1} \rangle$ is a **weak generalisation** of an operator Θ on fuzzy relations with the same signature, if for any total paraconsistent intuitionistic fuzzy relations R_1, \dots, R_n on schemes $\Sigma_1, \dots, \Sigma_n$, respectively, we have

$$\lambda_{\Sigma_{n+1}}(\Psi(R_1, \dots, R_n)) = \Theta(\lambda_{\Sigma_1}(R_1), \dots, \lambda_{\Sigma_n}(R_n)).$$

The above definition essentially requires Ψ to coincide with Θ on total paraconsistent intuitionistic fuzzy relations (which are in one-one correspondence with the fuzzy relations). In general, there may be many operators on paraconsistent intuitionistic fuzzy relations that are weak generalisations of a given operator Θ on fuzzy relations. The behavior of the weak generalisations of Θ on even just the consistent paraconsistent intuitionistic fuzzy relations may in general vary. We require a stronger notion of operator generalisation under which, when restricted to consistent paraconsistent intuitionistic fuzzy relations, the behavior of all the generalised operators is the same. Before we can develop such a notion, we need that of ‘representations’ of a paraconsistent intuitionistic fuzzy relation.

We associate with a consistent paraconsistent intuitionistic fuzzy relation R the set of all (fuzzy relations corresponding to) total paraconsistent intuitionistic fuzzy relations obtainable from R by filling in the gaps between the belief and doubt factors for each tuple. Let the map $\mathbf{reps}_\Sigma : \mathcal{C}(\Sigma) \rightarrow 2^{\mathcal{F}(\Sigma)}$ be given by

$$\mathbf{reps}_\Sigma(R) = \{Q \in \mathcal{F}(\Sigma) \mid \bigwedge_{t_i \in \tau(\Sigma)} (R(t_i)^+ \leq Q(t_i) \leq 1 - R(t_i)^-)\}.$$

The set $\mathbf{reps}_\Sigma(R)$ contains all fuzzy relations that are ‘completions’ of the consistent or pseudo-consistent paraconsistent intuitionistic fuzzy relation R . Observe that \mathbf{reps}_Σ is defined only for consistent paraconsistent intuitionistic fuzzy relations and produces sets of fuzzy relations. Then we have following observation.

Proposition 1. *For any consistent paraconsistent intuitionistic fuzzy relation R on scheme Σ , $\mathbf{reps}_\Sigma(R)$ is the singleton $\{\lambda_\Sigma(R)\}$ iff R is total.*

We now need to extend operators on fuzzy relations to sets of fuzzy relations. For any operator $\Theta : \mathcal{F}(\Sigma_1) \times \dots \times \mathcal{F}(\Sigma_n) \rightarrow \mathcal{F}(\Sigma_{n+1})$ on fuzzy relations, we let $\mathcal{S}(\Theta) : 2^{\mathcal{F}(\Sigma_1)} \times \dots \times 2^{\mathcal{F}(\Sigma_n)} \rightarrow 2^{\mathcal{F}(\Sigma_{n+1})}$ be a map on sets of fuzzy relations defined as follows. For any sets M_1, \dots, M_n of fuzzy relations on schemes $\Sigma_1, \dots, \Sigma_n$, respectively,

$$\mathcal{S}(\Theta)(M_1, \dots, M_n) = \{\Theta(R_1, \dots, R_n) \mid R_i \in M_i, \text{ for all } i, 1 \leq i \leq n\}.$$

In other words, $\mathcal{S}(\Theta)(M_1, \dots, M_n)$ is the set of Θ -images of all tuples in the cartesian product $M_1 \times \dots \times M_n$. We are now ready to lead up to a stronger notion of operator generalisation.

Definition 6. *An operator Ψ on paraconsistent intuitionistic fuzzy relations with signature $\langle \Sigma_1, \dots, \Sigma_{n+1} \rangle$ is **consistency preserving** if for any consistent or pseudo-consistent paraconsistent intuitionistic fuzzy relations R_1, \dots, R_n on schemes $\Sigma_1, \dots, \Sigma_n$, respectively, $\Psi(R_1, \dots, R_n)$ is also consistent or pseudo-consistent.*

Definition 7. *A consistency preserving operator Ψ on paraconsistent intuitionistic fuzzy relations with signature $\langle \Sigma_1, \dots, \Sigma_{n+1} \rangle$ is a **strong generalisation** of an operator Θ on fuzzy relations with the same signature, if for any consistent or pseudo-consistent paraconsistent intuitionistic fuzzy relations R_1, \dots, R_n on schemes $\Sigma_1, \dots, \Sigma_n$, respectively, we have*

$$\mathbf{reps}_{\Sigma_{n+1}}(\Psi(R_1, \dots, R_n)) = \mathcal{S}(\Theta)(\mathbf{reps}_{\Sigma_1}(R_1), \dots, \mathbf{reps}_{\Sigma_n}(R_n)).$$

Given an operator Θ on fuzzy relations, the behavior of a weak generalisation of Θ is ‘controlled’ only over the total paraconsistent intuitionistic fuzzy relations. On the other hand, the behavior of a strong generalisation is ‘controlled’ over all consistent or pseudo-consistent paraconsistent intuitionistic fuzzy relations. This itself suggests that strong generalisation is a stronger notion than weak generalisation. The following proposition makes this precise.

Proposition 2. *If Ψ is a strong generalisation of Θ , then Ψ is also a weak generalisation of Θ .*

Proof. Let $\langle \Sigma_1, \dots, \Sigma_{n+1} \rangle$ be the signature of Ψ and Θ , and let R_1, \dots, R_n be any total paraconsistent intuitionistic fuzzy relations on schemes $\Sigma_1, \dots, \Sigma_n$, respectively. Since all total relations are consistent, and Ψ is a strong generalisation of Θ , we have that

$$\mathbf{reps}_{\Sigma_{n+1}}(\Psi(R_1, \dots, R_n)) = \mathcal{S}(\Theta)(\mathbf{reps}_{\Sigma_1}(R_1), \dots, \mathbf{reps}_{\Sigma_n}(R_n)),$$

Proposition 1 gives us that for each i , $1 \leq i \leq n$, $\mathbf{reps}_{\Sigma_i}(R_i)$ is the singleton set $\{\lambda_{\Sigma_i}(R_i)\}$. Therefore, $\mathcal{S}(\Theta)(\mathbf{reps}_{\Sigma_1}(R_1), \dots, \mathbf{reps}_{\Sigma_n}(R_n))$ is just the singleton set: $\{\Theta(\lambda_{\Sigma_1}(R_1), \dots, \lambda_{\Sigma_n}(R_n))\}$. Here, $\Psi(R_1, \dots, R_n)$ is total, and $\lambda_{\Sigma_{n+1}}(\Psi(R_1, \dots, R_n)) = \Theta(\lambda_{\Sigma_1}(R_1), \dots, \lambda_{\Sigma_n}(R_n))$, i.e. Ψ is a weak generalisation of Θ . \square

Though there may be many strong generalisations of an operator on fuzzy relations, they all behave the same when restricted to consistent or pseudo-consistent paraconsistent intuitionistic fuzzy relations. In the next section, we propose strong generalisations for the usual operators on fuzzy relations. The proposed generalised operators on paraconsistent intuitionistic fuzzy relations correspond to the belief system intuition behind paraconsistent intuitionistic fuzzy relations.

First we will introduce two special operators on paraconsistent intuitionistic fuzzy relations called “split” and “combine” to transform inconsistent paraconsistent intuitionistic fuzzy relations into pseudo-consistent paraconsistent intuitionistic fuzzy relations and transform pseudo-consistent paraconsistent intuitionistic fuzzy relations into inconsistent paraconsistent intuitionistic fuzzy relations, respectively.

Definition 8 (Split). *Let R be a paraconsistent intuitionistic fuzzy relation on scheme Σ . Then,*

$$\Delta(R) = \{\langle t, b, d \rangle \mid \langle t, b, d \rangle \in R \text{ and } b + d \leq 1\} \cup \{\langle t, b', d' \rangle \mid (\exists b)(\exists d)(\langle t, b, d \rangle \in R \text{ and } b + d > 1 \text{ and } b' = b \text{ and } d' = 1 - b)\} \cup \{\langle t, b', d' \rangle \mid (\exists b)(\exists d)(\langle t, b, d \rangle \in R \text{ and } b + d > 1 \text{ and } b' = 1 - d \text{ and } d' = d)\}.$$

Definition 9 (Combine). *Let R be a paraconsistent intuitionistic fuzzy relation on scheme Σ . Then,*

$$\nabla(R) = \{\langle t, b, d \rangle \mid (\exists b')(\exists d')((\langle t, b', d' \rangle \in R \text{ and } (\forall b_i)(\forall d_i)(\langle t, b_i, d_i \rangle \in R \rightarrow d \geq d_i)) \text{ and } (\langle t, b, d' \rangle \in R \text{ and } (\forall b_i)(\forall d_i)(\langle t, b_i, d_i \rangle \in R \rightarrow b \geq b_i)))\}.$$

Note that strong generalization defined above only holds for consistent or pseudo-consistent paraconsistent intuitionistic fuzzy relations. For any paraconsistent intuitionistic fuzzy relations, we should first use split operation to transform them into consistent or pseudo-consistent paraconsistent intuitionistic fuzzy relations and apply the set-theoretic and relation-theoretic operations on them and finally use combine operation to transform the result into paraconsistent intuitionistic fuzzy relation. For the simplification of notation, the following generalized algebra is defined under such assumption.

3 Generalized Algebra on Paraconsistent Intuitionistic Fuzzy Relations

In this section, we present one strong generalisation each for the fuzzy relation operators such as union, join, projection. To reflect generalisation, a hat

is placed over a fuzzy relation operator to obtain the corresponding paraconsistent intuitionistic fuzzy relation operator. For example, \bowtie denotes the natural join among fuzzy relations, and $\hat{\bowtie}$ denotes natural join on paraconsistent intuitionistic fuzzy relations. These generalized operators maintain the belief system intuition behind paraconsistent intuitionistic fuzzy relations.

Set-Theoretic Operators

We first generalize the two fundamental set-theoretic operators, union and complement.

Definition 10. *Let R and S be paraconsistent intuitionistic fuzzy relations on scheme Σ . Then,*

- (a) *the **union** of R and S , denoted $R \hat{\cup} S$, is a paraconsistent intuitionistic fuzzy relation on scheme Σ , given by*

$$(R \hat{\cup} S)(t) = \langle \max\{R(t)^+, S(t)^+\}, \min\{R(t)^-, S(t)^-\} \rangle, \text{ for any } t \in \tau(\Sigma);$$

- (b) *the **complement** of R , denoted $\hat{\complement} R$, is a paraconsistent intuitionistic fuzzy relation on scheme Σ , given by*

$$(\hat{\complement} R)(t) = \langle R(t)^-, R(t)^+ \rangle, \text{ for any } t \in \tau(\Sigma).$$

Proposition 3. *The operators $\hat{\cup}$ and unary $\hat{\complement}$ on paraconsistent intuitionistic fuzzy relations are strong generalisations of the operators \cup and unary $-$ on fuzzy relations.*

Definition 11. *Let R and S be paraconsistent intuitionistic fuzzy relations on scheme Σ . Then,*

- (a) *the **intersection** of R and S , denoted $R \hat{\cap} S$, is a paraconsistent intuitionistic fuzzy relation on scheme Σ , given by*

$$(R \hat{\cap} S)(t) = \langle \min\{R(t)^+, S(t)^+\}, \max\{R(t)^-, S(t)^-\} \rangle, \text{ for any } t \in \tau(\Sigma);$$

- (b) *the **difference** of R and S , denoted $R \hat{\ominus} S$, is a paraconsistent intuitionistic fuzzy relation on scheme Σ , given by*

$$(R \hat{\ominus} S)(t) = \langle \min\{R(t)^+, S(t)^-\}, \max\{R(t)^-, S(t)^+\} \rangle, \text{ for any } t \in \tau(\Sigma);$$

Relation-Theoretic Operators

We now define some relation-theoretic algebraic operators on paraconsistent intuitionistic fuzzy relations.

Definition 12. *Let R and S be paraconsistent intuitionistic fuzzy relations on schemes Σ and Δ , respectively. Then, the **natural join** (further for short called join) of R and S , denoted $R \hat{\bowtie} S$, is a paraconsistent intuitionistic fuzzy relation on scheme $\Sigma \cup \Delta$, given by*

$$(R \hat{\bowtie} S)(t) = \langle \min\{R(\pi_\Sigma(t))^+, S(\pi_\Delta(t))^+\}, \max\{R(\pi_\Sigma(t))^-, S(\pi_\Delta(t))^- \}\rangle,$$

where π is the usual projection of a tuple.

It is instructive to observe that, similar to the intersection operator, the minimum of the belief factors and the maximum of the doubt factors are used in the definition of the join operation.

Proposition 4. $\hat{\bowtie}$ is a strong generalisation of \bowtie .

Definition 13. Let R be a paraconsistent intuitionistic fuzzy relation on scheme Σ , and $\Delta \subseteq \Sigma$. Then, the **projection** of R onto Δ , denoted $\hat{\pi}_\Delta(R)$, is a paraconsistent intuitionistic fuzzy relation on scheme Δ , given by

$$(\hat{\pi}_\Delta(R))(t) = \langle \max\{R(u)^+ | u \in t^\Sigma\}, \min\{R(u)^- | u \in t^\Sigma\}\rangle.$$

The belief factor of a tuple in the projection is the maximum of the belief factors of all of the tuple’s extensions onto the scheme of the input paraconsistent intuitionistic fuzzy relation. Moreover, the doubt factor of a tuple in the projection is the minimum of the doubt factors of all of the tuple’s extensions onto the scheme of the input paraconsistent intuitionistic fuzzy relation.

Definition 14. Let R be a paraconsistent intuitionistic fuzzy relation on scheme Σ , and let F be any logic formula involving attribute names in Σ , constant symbols (denoting values in the attribute domains), equality symbol $=$, negation symbol \neg , and connectives \vee and \wedge . Then, the **selection** of R by F , denoted $\hat{\sigma}_F(R)$, is a paraconsistent intuitionistic fuzzy relation on scheme Σ , given by

$$(\hat{\sigma}_F(R))(t) = \langle \alpha, \beta \rangle, \text{ where}$$

$$\alpha = \begin{cases} R(t)^+ & \text{if } t \in \sigma_F(\tau(\Sigma)) \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad \beta = \begin{cases} R(t)^- & \text{if } t \in \sigma_F(\tau(\Sigma)) \\ 1 & \text{otherwise} \end{cases}$$

where σ_F is the usual selection of tuples satisfying F from ordinary relations.

If a tuple satisfies the selection criterion, its belief and doubt factors are the same in the selection as in the input paraconsistent intuitionistic fuzzy relation. In the case where the tuple does not satisfy the selection criterion, its belief factor is set to 0 and the doubt factor is set to 1 in the selection.

Proposition 5. The operators $\hat{\pi}$ and $\hat{\sigma}$ are strong generalisations of π and σ , respectively.

4 Conclusions

We have presented a generalization of fuzzy relations and paraconsistent relations, called paraconsistent intuitionistic fuzzy relations, in which we allow the

representation of confidence (belief and doubt) factors with each tuple. The algebra on fuzzy relations is appropriately generalized to manipulate paraconsistent intuitionistic fuzzy relations.

Our data model can be used to represent relational information that may be incomplete or inconsistent. As usual, the algebraic operators can be used to construct queries to any database systems for retrieving imprecise information.

References

1. Codd, E.: A relational model for large shared data banks. *Communications of the ACM* **13** (1970) 377–387
2. Brodie, M.L., Mylopoulos, J., Schmidt, J.W.: On the development of data models. *On Conceptual Modelling* (1984) 19–47
3. Biskup, J.: A foundation of codd's relational maybe-operations. *ACM Trans. Database Syst.* **8** **4** (1983) 608–636
4. Imieliński, T., Lipski, W.: Incomplete information in relational databases. *Journal of the ACM* **31** (1984) 761–791
5. Codd, E.: Extending the database relational model to capture more meaning. *ACM Transactions of Database Systems* **4** (1979) 397–434
6. Zadeh, L.A.: Fuzzy sets. *Inf. Control* **8** (1965) 338–353
7. Turksen, I.: Interval valued fuzzy sets based on normal forms. *Fuzzy Sets and Systems* **20** (1986) 191–210
8. Atanassov, K.: Intuitionistic fuzzy sets. *Fuzzy Sets and Systems* **20** (1986) 87–96
9. Bosc, P., Prade, H.: An introduction to fuzzy set and possibility theory-based approaches to treatment of uncertainty and imprecision in database management systems. In: *Proc. Workshop Uncertainty Management in Information Systems*. (1993)
10. Bosc, P., Pivert, O.: Sqlf: A relational database language for fuzzy querying. *IEEE Transactions on Fuzzy Systems* **3** (1995)
11. Kacprzyk, J., Ziolkowski, A.: Database queries with fuzzy linguistic quantifiers. *IEEE Trans. Syst. Man Cybern. SMC-16* **3** (1986) 474–479
12. Prade, H., Testemale, C.: Representation of soft constraints and fuzzy attribute values by means of possibility distributions in databases. *Analysis of Fuzzy Information, Volume II, Artificial Intelligence and Decision Systems* (1987) 213–229
13. Raju, K.V.S.V.N., Majumdar, A.K.: Fuzzy functional dependencies and lossless join decomposition of fuzzy relational database systems. *ACM Trans. on Database Syst.* **13** **2** (1988) 129–166
14. Vila, M., Cubero, J., Medina, J., Pons, O.: Logic and fuzzy relational databases: a new language and a new definition. In Bosc, P., Kacprzyk, J., eds.: *Fuzziness in Database Management Systems*. (Physica-Verlag)
15. de Amo, S., Carnielli, W., Marcos, J.: A logical framework for integrating inconsistent information in multiple databasses. In: *Proc. PoIKS'02, LNCS 2284*. (2002) 67–84
16. Bagai, R., Sunderraman, R.: A paraconsistent relational data model. *International Journal of Computer Mathematics* **55** (1995)