# Semantic Web/DL Practice Problems Solutions

## Problem 1

Consider the base concepts: ***Person, Happy, Animal, Cat, Old, Fish*** and the role ***Owns(Person,Thing)***

Formulate ALC concepts for the following:

1. **happy person**. Person ⊓ Happy
2. **happy pet owner**. Person ⊓ Happy ⊓ ∃ owns.Animal
3. **person who owns only cats**. Person ⊓ ∀ owns.Cat
4. **unhappy pet owners who own an old cat**. Person ⊓ ¬ Happy ⊓ ∃ owns.(Animal ⊓ Cat ⊓ Old)
5. **pet owners who only own cats and fish**. Person ⊓ ∃ owns.Animal ⊓ ∀ owns.(Cat ⊔ Fish)

## Problem 2

For each of the following axioms given below, determine which of the three interpretations ($I_1$ , $I_2$ , and $I_3$ ) in the subsequent table satisfy it. Assume A, B, C, and D are atomic concepts and P is a role.

1. B ⊑ D
2. A ⊑ B ⊓ ∀ P.C
3. D ⊑ B ⊔ ∃ P.C

| **Classes/Roles** | $I_1$ | $I_2$ | $I_3$ |
|---:|---:|---:|---:|
| $A^I$ | {} | { a } | { b, c } |
| $B^I$ | { a, b } | { a } | { b, c, d } |
| $C^I$ | { b } | { b, d } | { a, b } |
| $D^I$ | { } | { a, b } | { a, b, c, d } |
| $P^I$ | { } | { (a,b), (a,c), (b,d) } | { (b,a), (b,b), (d,a) } |

SOLUTION:

| Formula | $I_1$ | $I_2$ | $I_3$ |
|---|---|---|---|
| B ⊑ D | No | Yes | Yes |
| A ⊑ B ⊓ ∀ P.C | Yes | No | Yes |
| D ⊑ B ⊔ ∃ P.C | Yes | Yes | No |

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## Problem 3

Show that the following KB is unsatisfiable.

**TBox**

MixedTeam ≡ Team ⊓ ∃ hasMember.Male ⊓ ∃ hasMember.Female

Male ≡ ¬ Female

**ABox**

MixedTeam(fc)

(∀ hasMember.Male)(fc)

SOLUTION:

After pre-processing, we get the initial ABox as follows:

$A_0$ = { (Team ⊓ ∃ hasMember.¬ Female ⊓ ∃ hasMember.Female)(fc), (∀ hasMember.¬ Female)(fc) }

Using the ⊓-rule, we can ADD the following to $A_0$:

(∃ hasMember.Female)(fc)

Using ∃-rule on the added fact, we can ADD the following:

hasMember(fc,a)

Female(a)

Applying the ∀-rule on (∀ hasMember.¬ Female)(fc) and hasMember(fc,a), we can ADD:

¬ Female(a)

arriving at a CLASH.

Hence, the KB is unsatisfiable.

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**Problem 4**

Show that the following KB is unsatisfiable.

**TBox**

Woman ≡ Person ⊓ Female

**ABox**

Person(ann)

Female(ann)

¬ Woman(ann)

SOLUTION:

After pre-processing, we get the following ABox:

$A_0$ = { Person(ann), Female(ann), (¬ Person ⊔ ¬ Female)(ann) }

Applying the ⊔-rule, we get:

$A_1$ = { Person(ann), Female(ann), (¬ Person(ann) ⊔ ¬ Female(ann)) }

Applying the ⊔-rule, we get:

$A_{20}$ = { Person(ann), Female(ann), ¬ Person(ann) } containing a CLASH

$A_{21}$ = { Person(ann), Female(ann), ¬ Female(ann) } containing a CLASH

Hence the KB is unsatisfiable.

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