Penguin Problem

Consider the KB:

TBox:

Bird $\sqsubseteq$ Flies

Penguin $\sqsubseteq$ Bird

Penguin $\sqcap$ Flies $\sqsubseteq$ $\bot$

Abox:

Penguin(tweety)

QUESTION

Prove that the KB is **unsatisfiable**

SOLUTION

Let us abbreviate Bird as b, Penguin as p, Flies by f, and tweety as t.

The three TBox axioms, when converted to "single concept" form, gives us the following:

$(\neg b \sqcup f)$

$(\neg p \sqcup b)$

$(\neg p \sqcup \neg f)$

Since there is only one "individual", t, the initial ABOX after pre-processing and instantiating the single concepts is the following:

A0 = { p(t), $(\neg b \sqcup f)(t)$, $(\neg p \sqcup b)(t)$, $(\neg p \sqcup \neg f)(t)$ }

Applying the $\sqcup$ rule three times, we get the following 8 ABoxes:

A1 = { **p(t)**, ¬b(t), **¬p(t)** }

A2 = { **p(t)**, ¬b(t), **¬p(t)**, ¬f(t) }

A3 = { p(t), ¬**b(t)**, **b(t)**, ¬p(t) }

A4 = { p(t), ¬**b(t)**, **b(t)**, ¬f(t) }

A5 = { **p(t)**, f(t), ¬**p(t)** }

A6 = { **p(t)**, f(t), ¬**p(t)**, ¬f(t) }

A7 = { **p(t)**, f(t), b(t), ¬**p(t)** }

A8 = { p(t), **f(t)**, b(t), ¬**f(t)** }

We see that there is a clash in each and every one of the 8 ABoxes. So, the KB is unsatisfiable