Harry Potter Tableaux Problem

Given knowledge base K:

TBox:

Human $\sqsubseteq$ $\exists$hasParent.Human

Orphan $\sqsubseteq$ Human $\sqcap$ $\forall$hasParent.$\neg$Alive

Converting TBox Concept Inclusions into single concept form, we get

$\neg$Human $\sqcup$ $\exists$hasParent.Human

$\neg$Orphan $\sqcup$ (Human $\sqcap$ $\forall$hasParent.$\neg$Alive)

ABox:

Orphan(harrypotter), hasParent(harrypotter,jamespotter)

Using the tableaux method show that $\neg$Alive(jamespotter) is a logical consequence of K.

Let us include Alive(jamespotter) to the Abox and try to show K is unsatisfiable.

Also, let us abbreviate terms as follows:

term	abbreviation
Human	h
hasParent	hp
Orphan	o
Alive	a
harrypotter	h
jamespotter	j

INITIAL ABOX:

A0
o(h)
hp(h,j)
a(j)
($\neg$h $\sqcup$ $\exists$hp.h)(h)
($\neg$h $\sqcup$ $\exists$hp.h)(j)
($\neg$o $\sqcup$ (h $\sqcap$ $\forall$hp.$\neg$a))(h)
($\neg$o $\sqcup$ (h $\sqcap$ $\forall$hp.$\neg$a))(j)

Applying $\sqcup$-rule to **($\neg$o $\sqcup$ (h $\sqcap$ $\forall$hp.$\neg$a))(h)** gives:

A11 (INCONSISTENT)
o(h)
hp(h,j)
a(j)
$(\neg h \sqcup \exists hp.h)(h)$
$(\neg h \sqcup \exists hp.h)(j)$
$(\neg o \sqcup (h \sqcap \forall hp.\neg a))(h)$
$(\neg o \sqcup (h \sqcap \forall hp.\neg a))(j)$
¬o(h)

A12
o(h)
hp(h,j)
a(j)
$(\neg h \sqcup \exists hp.h)(h)$
$(\neg h \sqcup \exists hp.h)(j)$
$(\neg o \sqcup (h \sqcap \forall hp.\neg a))(h)$
$(\neg o \sqcup (h \sqcap \forall hp.\neg a))(j)$
$\mathbf{(h \sqcap \forall hp.\neg a)(h)}$

A11 contains a clash and is inconsistent; So, A11 can be discarded and not expanded further.

Applying $\sqcap$*-rule to* $\mathbf{(h \sqcap \forall hp.\neg a)(h)}$ gives:

A2
o(h)
hp(h,j)
a(j)
$(\neg h \sqcup \exists hp.h)(h)$
$(\neg h \sqcup \exists hp.h)(j)$
$(\neg o \sqcup (h \sqcap \forall hp.\neg a))(h)$
$(\neg o \sqcup (h \sqcap \forall hp.\neg a))(j)$
$(h \sqcap \forall hp.\neg a)(h)$
h(h)
$\mathbf{(\forall hp.\neg a)(h)}$

Applying $\sqcup$*-rule to* $\mathbf{(\neg h \sqcup \exists hp.h)(h)}$ gives:

A31 (INCONSISTENT)
o(h)

A31 (INCONSISTENT)
hp(h,j)
a(j)
$(\neg h \sqcup \exists hp.h)(h)$
$(\neg h \sqcup \exists hp.h)(j)$
$(\neg o \sqcup (h \sqcap \forall hp.\neg a))(h)$
$(\neg o \sqcup (h \sqcap \forall hp.\neg a))(j)$
$(h \sqcap \forall hp.\neg a)(h)$
h(h)
$(\forall hp.\neg a)(h)$
¬h(h)

A32
o(h)
hp(h,j)
a(j)
$(\neg h \sqcup \exists hp.h)(h)$
$(\neg h \sqcup \exists hp.h)(j)$
$(\neg o \sqcup (h \sqcap \forall hp.\neg a))(h)$
$(\neg o \sqcup (h \sqcap \forall hp.\neg a))(j)$
$(h \sqcap \forall hp.\neg a)(h)$
h(h)
$(\forall hp.\neg a)(h)$
(∃hp.h)(h)

A31 contains a clash and is inconsistent; So, A31 can be discarded and not expanded further.

Applying ∀*-rule to* $(\forall hp.\neg a)(h)$ gives:

A4 (INCONSISTENT)
o(h)
hp(h,j)
a(j)
$(\neg h \sqcup \exists hp.h)(h)$
$(\neg h \sqcup \exists hp.h)(j)$
$(\neg o \sqcup (h \sqcap \forall hp.\neg a))(h)$
$(\neg o \sqcup (h \sqcap \forall hp.\neg a))(j)$
$(h \sqcap \forall hp.\neg a)(h)$
h(h)
$(\forall hp.\neg a)(h)$
$(\exists hp.h)(h)$

A4 (INCONSISTENT)
¬a(j)

All Aboxes have ended up with clashes! So, KB with a(j) is unsatisfiable; Hence ¬a(j) is a logical consequence of K.