

STEP BY STEP Application of rules in the Tableaux algorithm for the example in the slides:

After initial pre-processing steps, we get:

$$A^2 = \{ (\exists R.A)(a), (\forall R.B)(a), (\forall R.(\neg A \sqcup \neg B))(a) \}$$

Applying \exists -rule to $(\exists R.A)(a)$ gives:

$$A^3 = \{ (\exists R.A)(a), (\forall R.B)(a), (\forall R.(\neg A \sqcup \neg B))(a), \mathbf{R(a,b)}, \mathbf{A(b)} \}$$

Applying \forall -rule to $(\forall R.B)(a)$ gives:

$$A^4 = \{ (\exists R.A)(a), (\forall R.B)(a), (\forall R.(\neg A \sqcup \neg B))(a), R(a,b), A(b), \mathbf{B(b)} \}$$

Applying \forall -rule to $(\forall R.(\neg A \sqcup \neg B))(a)$ gives:

$$A^5 = \{ (\exists R.A)(a), (\forall R.B)(a), (\forall R.(\neg A \sqcup \neg B))(a), R(a,b), A(b), B(b), (\neg \mathbf{A} \sqcup \neg \mathbf{B})(b) \}$$

Applying \sqcup -rule to $(\neg A \sqcup \neg B)(b)$ gives the following two interpretations:

$$A^{61} = \{ (\exists R.A)(a), (\forall R.B)(a), (\forall R.(\neg A \sqcup \neg B))(a), R(a,b), \mathbf{A(b)}, B(b), (\neg A \sqcup \neg B)(b), \neg \mathbf{A(b)} \}$$

$$A^{62} = \{ (\exists R.A)(a), (\forall R.B)(a), (\forall R.(\neg A \sqcup \neg B))(a), R(a,b), A(b), \mathbf{B(b)}, (\neg A \sqcup \neg B)(b), \neg \mathbf{B(b)} \}$$

Both these interpretations contain "clashes"; So, the original KB is UNSATISFIABLE!