CSc 8710, Fall 2016, Homework 1, Due: September 4, 2016 (Sunday)

1. Consider the following Propositional Logic sentence:

$$\neg(P \wedge \neg(Q \vee R)) \leftrightarrow (\neg P \vee (R \vee Q))$$

   (a) Make a truth table for this sentence.

   (b) Is the sentence a tautology? a contradiction? a contingency?

   (c) If it is a tautology, prove that an argument with an empty premise and the sentence as the conclusion is valid.

   (d) If it is a contradiction, prove that an argument with an empty premise and the negation of the sentence as the conclusion is valid.

   (e) If it is a contingency, provide an interpretation that makes the sentence true and another interpretation that makes the sentence false. sentence as the conclusion is valid.

2. Use logical equivalences to simplify the following sentence:

$$((P \rightarrow (\neg P)) \vee ((\neg Q \rightarrow Q)) \wedge ((\neg P) \rightarrow Q)$$

3. Provide a proof for the following argument:

Premises:
1. $\neg A \rightarrow B$
2. $C \wedge (D \rightarrow E)$
3. $(B \wedge \neg G) \rightarrow (C \rightarrow \neg E)$
4. $\neg A \wedge \neg H$
5. $\neg A \rightarrow \neg G$

Conclusion: $\neg E$

4. Let I be an interpretation over the *non-negative integers* under which :

$a \leftarrow 0$

$x \leftarrow 1$

$f \leftarrow f_I$ where $f_I(d) = d + 1$

$p \leftarrow p_I$ where $p_I(d1, d2) =$ "$d1 < d2$"

Determine the truth value of each of the following sentences under I :

(a) $p(a, x) \wedge p(x, f(x))$

(b) $\exists y(p(y, a) \vee p(f(y), y))$

(c) $\forall x \exists y p(x, y)$

(d) $\exists y \forall x p(x, y)$

5. Consider the following argument:

Premises:
1. Every logic student with a heart likes Steve.
2. There are logic students.
3. Everybody has a heart.
Conclusion: Somebody likes Steve.

   (a) Translate the argument into First Order Logic, clearly indicating the meaning of predicate symbols you choose.
   (b) Provide an interpretation for the argument that makes every premise and the conclusion true.
   (c) Provide a completed proof for the argument.

6. Provide a proof for the argument:

Premises:
1. $(\exists x)(A(x) \wedge \neg B(x))$
2. $(\forall x)(\neg C(x) \vee D(x))$
3. $(\forall x)(D(x) \rightarrow B(x))$
Conclusion: $(\exists x)(A(x) \wedge \neg C(x))$

7. Consider the following argument:

Premises:
1. $(\forall x)(G(x) \rightarrow B(x))$
2. $(\forall x)(B(x) \rightarrow H(x))$
3. $(\exists x)H(x)$
Conclusion: $(\exists x)G(x)$

   Provide an interpretation that demonstrates that the argument is invalid.

8. Provide a proof for the argument:

Premises:
1: $(\forall x)(\forall y)(F(x,y) \leftrightarrow F(y,x))$
2: $(\forall x)(\forall y)(\forall z)((F(x,y) \wedge F(y,z)) \rightarrow F(x,z))$
3: $(\exists x)(\exists y)F(x,y)$
Conclusion: $(\forall x)F(x,x)$