1. Use the truth table method to find which of the following sentences are valid, satisfiable, or contradictory:
   
   (a) \((P \rightarrow (Q \rightarrow R)) \leftrightarrow ((P \land Q) \rightarrow R)\)
   
   (b) \((P \rightarrow (Q \lor R)) \leftrightarrow ((P \land (\neg Q)) \rightarrow R)\)

2. Use logical equivalences to simplify each of the following sentences:
   
   (a) \(\neg ((P \rightarrow P) \land (P \rightarrow Q)) \land (Q \rightarrow (\neg P))\)
   
   (b) \((P \rightarrow (\neg P)) \lor ((\neg Q \rightarrow Q)) \land ((\neg P) \rightarrow Q)\)

3. Consider the argument: If I study or if I am a genius, then I will pass the course. If I pass the course, then I will be allowed to take the next course. But, I am not allowed to take the next course. Therefore, I am not a genius.

   (a) Write a theorem for the above argument.
   
   (b) Provide a formal proof for the theorem obtained in (a).

4. Let I be an interpretation over the non-negative integers under which:

   \[ a \leftarrow 0 \]
   \[ x \leftarrow 1 \]
   \[ f \leftarrow f_I \text{ where } f_I(d) = d + 1 \]
   \[ p \leftarrow p_I \text{ where } p_I(d_1, d_2) = "d_1 < d_2" \]

   Determine the truth value of each of the following sentences under I:

   (a) \(p(a, x) \land p(x, f(x))\)
   
   (b) \(\exists y (p(y, a) \lor p(f(y), y))\)
   
   (c) \(\forall x \exists y p(x, y)\)
   
   (d) \(\exists y \forall x p(x, y)\)

5. Prove each of the following:

   (a) \(\neg \exists x p(x) \iff \forall x (\neg p(x))\)
   
   (b) \(\exists x (p(x) \lor q(x)) \Rightarrow \exists x p(x) \lor \exists x q(x)\)

6. Disprove each of the following:

   (a) \(\exists x p(x) \Rightarrow \forall x p(x)\)
   
   (b) \(\forall x (p(x) \lor q(x)) \Rightarrow (\forall x p(x) \lor \forall x q(x))\)

7. Construct a formal proof for the following theorem:

   \(H1: (\exists x)(p(x) \land \neg r(x))\)
   
   \(H2: (\exists x)p(x) \rightarrow (\forall x)r(x)\)
   
   \(C: (\forall x)r(x) \land (\neg (\forall x)r(x))\)