CSc 8710, Fall 2012, Homework 1, Due: Aug 29 (Wednesday)

1. Use the truth table method to find which of the following sentences are valid, satisfiable, or contradictory :

   **(a)** $(P \rightarrow (Q \rightarrow R)) \leftrightarrow ((P \wedge Q) \rightarrow R)$

   **(b)** $(P \rightarrow (Q \vee R)) \leftrightarrow ((P \wedge (\neg Q)) \rightarrow R)$

2. Use logical equivalences to simplify each of the following sentences :

   **(a)** $\neg(((P \rightarrow P) \wedge (P \rightarrow Q)) \wedge (Q \rightarrow (\neg P)))$

   **(b)** $((P \rightarrow (\neg P)) \vee ((\neg Q \rightarrow Q)) \wedge ((\neg P) \rightarrow Q)$

3. Consider the argument : If I study or if I am a genius, then I will pass the course. If I pass the course, then I will be allowed to take the next course. But, I am not allowed to take the next course. Therefore, I am not a genius.

   **(a)** Write a theorem for the above argument.

   **(b)** Provide a formal proof for the theorem obtained in (a).

4. Let I be an interpretation over the *non-negative integers* under which :

   $a \leftarrow 0$

   $x \leftarrow 1$

   $f \leftarrow f_I$ where $f_I(d) = d + 1$

   $p \leftarrow p_I$ where $p_I(d1, d2) =$ "$d1 < d2$"

   Determine the truth value of each of the following sentences under I :

   **(a)** $p(a, x) \wedge p(x, f(x))$

   **(b)** $\exists y(p(y, a) \vee p(f(y), y))$

   **(c)** $\forall x \exists y p(x, y)$

   **(d)** $\exists y \forall x p(x, y)$

5. Prove each of the following :

   **(a)** $\neg\exists x p(x) \Leftrightarrow \forall x(\neg p(x))$

   **(b)** $\exists x(p(x) \vee q(x)) \Rightarrow \exists x p(x) \vee \exists x q(x)$

6. Disprove each of the following :

   **(a)** $\exists x p(x) \Rightarrow \forall x p(x)$

   **(b)** $\forall x(p(x) \vee q(x)) \Rightarrow (\forall x p(x) \vee \forall x q(x))$

7. Construct a formal proof for the following theorem:

$$\begin{array}{l} \text{H1: } (\exists x)(p(x) \wedge \neg r(x)) \\ \text{H2: } (\exists x)p(x) \rightarrow (\forall x)r(x) \\ \hline \text{C: } (\forall x)r(x) \wedge (\neg(\forall x)r(x)) \end{array}$$