CSc 8710 Homework 1

Answer

September 7, 2010

1

(5pt+5pt)

(a) $((P \wedge Q) \leftrightarrow P) \leftrightarrow (P \leftrightarrow Q)$ is satisfiable

$\Im_n$	P	Q	$(P \wedge Q)$	$((P \wedge Q) \leftrightarrow P)$	$(P \leftrightarrow Q)$	(a)
$\Im_1$	F	F	F	T	T	T
$\Im_2$	F	T	F	T	F	F
$\Im_3$	T	F	F	F	F	T
$\Im_4$	T	T	T	T	T	T

simplify: $(Q \rightarrow P)$

(b) $((P \rightarrow Q) \vee (R \rightarrow S)) \rightarrow ((P \vee R) \rightarrow (Q \vee S))$ is satisfiable

$\Im_n$	P	Q	R	S	$(P \rightarrow Q)$	$(R \rightarrow S)$	$(P \vee R)$	$(Q \vee S)$
$\Im_1$	F	F	F	F	T	T	F	F
$\Im_2$	F	F	F	T	T	T	F	T
$\Im_3$	F	F	T	F	T	F	T	F
$\Im_4$	F	F	T	T	T	T	T	T
$\Im_5$	F	T	F	F	T	T	F	T
$\Im_6$	F	T	F	T	T	T	F	T
$\Im_7$	F	T	T	F	T	F	T	T
$\Im_8$	F	T	T	T	T	T	T	T
$\Im_9$	T	F	F	F	F	T	T	F
$\Im_{10}$	T	F	F	T	F	T	T	T
$\Im_{11}$	T	F	T	F	F	F	T	F
$\Im_{12}$	T	F	T	T	F	T	T	T
$\Im_{13}$	T	T	F	F	T	T	T	T
$\Im_{14}$	T	T	F	T	T	T	T	T
$\Im_{15}$	T	T	T	F	T	F	T	T
$\Im_{16}$	T	T	T	T	T	T	T	T

$\Im_n$	$((P \to Q) \vee (R \to S))$	$((P \vee R) \to (Q \vee S))$	(b)
$\Im_1$	T	T	T
$\Im_2$	T	T	T
$\Im_3$	T	F	F
$\Im_4$	T	T	T
$\Im_5$	T	T	T
$\Im_6$	T	T	T
$\Im_7$	T	T	T
$\Im_8$	T	T	T
$\Im_9$	T	F	F
$\Im_{10}$	T	T	T
$\Im_{11}$	F	F	T
$\Im_{12}$	T	T	T
$\Im_{13}$	T	T	T
$\Im_{14}$	T	T	T
$\Im_{15}$	T	T	T
$\Im_{16}$	T	T	T

2

(10pt)

$((P \wedge R) \to Q) \wedge ((P \wedge (\neg R)) \to Q)$	Given
$((\neg(P \wedge R)) \vee Q) \wedge ((\neg(P \wedge (\neg R))) \vee Q)$	E12 (twice)
$((\neg P) \vee (\neg R) \vee Q) \wedge ((\neg P) \wedge (\neg(\neg R)) \wedge Q)$	E8
$((\neg P) \vee (\neg R) \vee Q) \wedge ((\neg P) \wedge R \wedge Q)$	E11
$((\neg P) \vee Q \vee (\neg R)) \wedge ((\neg P) \wedge Q \wedge R)$	E3
$((\neg P) \vee Q) \vee ((\neg R) \wedge R)$	E10
$((\neg P) \vee Q) \vee (R \wedge ((\neg R))$	E4
$((\neg P) \vee Q) \vee 0$	E22
$(\neg P) \vee Q$	E19
$P \to Q$	E12

3

(10pt)

1	$(P \wedge Q) \to R$	H1
2	$(\neg Q) \to (\neg S)$	H2
3	$\neg(P \wedge Q) \vee R$	E12, 1
4	$((\neg P) \vee (\neg Q)) \vee R$	E8, 3
5	$((\neg P) \vee R) \vee (\neg Q)$	E3, 4
6	$((\neg P) \vee R) \vee (\neg S)$	H2, 5
7	$(R \vee (\neg S)) \vee (\neg P)$	E3, 6
8	$((\neg R) \wedge S) \to (\neg P)$	E12, 7

4

(5pt+10pt)
(a)

H1:	$(F \vee A) \to M$
H2:	$M \to P$
H3:	$\neg P$
C:	$\neg A$

(b)

1	$(M \to P)$	H2
2	$\neg P$	H3
3	$\neg M$	I6, 2 and 1
4	$\neg(F \vee A)$	I6, 3 and H1
5	$(\neg F) \wedge (\neg A)$	E7, 4
6	$\neg A$	E4,I4, 5

5

(10pt)

1	$(\exists x(p(x) \wedge \neg r(x)))$	H1
2	$\exists x(\neg r(x))$	E4, I4, 1
3	$\neg \forall x r(x)$	E24, 2
4	$\exists x p(x)$	I4, 1
5	$\forall x r(x)$,	I5, H2, 4
6	$(\forall x r(x)) \wedge (\neg(\forall x r(x)))$	3 and 5

6

(5pt+10pt)
(a)

H1:	$\forall x(S(x) \to T(x))$
H2:	$\forall x(T(x) \to C(x))$
H3:	$\neg C(\text{Sam})$
C:	$\neg \forall x S(x)$

(b)

1	$\forall x(S(x) \rightarrow T(x))$	H1
2	$\forall x(T(x) \rightarrow C(x))$	H2
3	$\forall x((S(x) \rightarrow T(x)) \wedge (T(x) \rightarrow C(x))$	E26, 1 and 2
4	$\forall x(S(x) \rightarrow C(x))$	I8, 3
5	$S(\text{Sam}) \rightarrow C(\text{Sam})$	I16, 4
6	$\neg S(\text{Sam})$	I6, 5
7	$\exists x(\neg S(x))$	I17, 6
8	$\neg \forall x S(x)$	E24, 7

7

(5pt+5pt)
(a) is satisfiable (valid).
p(x) : x is divisible by 4
q(x): x is even number
$\forall x(p(x) \rightarrow q(x))$ is true.
$\forall x p(x)$ is false, and $\forall x q(x)$ is false; so $\forall x p(x) \rightarrow \forall x q(x)$ is true.
Thus, the statment is true in this example.

p(x) : x is even number
q(x) : x is divisible by 4
$\forall x(p(x) \rightarrow q(x))$ is false.
Thus, the statment is true in this example.

(b) is satisfiable.
p(x) : x is divisible by 4
q(x): x is even number
$\forall x p(x)$ is false, and $\forall x q(x)$ is false; so $\forall x p(x) \rightarrow \forall x q(x)$ is true.
$\forall x(p(x) \rightarrow q(x))$ is true.
Thus, the statment is true in this example.

p(x) : x is even number
q(x) : x is divisible by 4
$\forall x p(x)$ is false, and $\forall x q(x)$ is false; so $\forall x p(x) \rightarrow \forall x q(x)$ is true.
$\forall x(p(x) \rightarrow q(x))$ is false.
Thus, the statment is false in this example.