CSc 8710 Deductive Databases and Logic Programming
Homework 1
Fall 2010
Due: September 1, 2010 (Wednesday)

1. Determine whether the following propositional logic formulas are valid or satisfiable or contradictions.

   (a) $((P \wedge Q) \leftrightarrow P) \leftrightarrow (P \leftrightarrow Q)$

   (b) $((P \rightarrow Q) \vee (R \rightarrow S)) \rightarrow ((P \vee R) \rightarrow (Q \vee S))$

2. Simplify the following propositional logic sentence:

$$((P \wedge R) \rightarrow Q) \wedge ((P \wedge (\neg R)) \rightarrow Q)$$

   The goal is to obtain an equivalent sentence which has no more than one logical connective.

3. Construct a proof for the following theorem:

$$\begin{array}{l} \text{H1: } (P \wedge Q) \rightarrow R \\ \text{H2: } (\neg Q) \rightarrow (\neg S) \\ \hline \text{C: } ((\neg R) \wedge S) \rightarrow (\neg P) \end{array}$$

4. Consider the following argument:

   > If horses fly or cows eat artichokes, then the mosquito is the national bird. If the mosquito is the national bird, then peanut butter tastes good on hot dogs. But, peanut butter tastes terrible on hot dogs. Therefore, cows do not eat artichokes.

   (a) Using the propositional symbols F, A, M, P state the argument as a theorem.

   (b) Provide a proof for the theorem.

5. Construct a formal proof for the following theorem:

$$\begin{array}{l} \text{H1: } (\exists x(p(x) \wedge \neg r(x))) \\ \text{H2: } (\exists x p(x)) \rightarrow (\forall x r(x)) \\ \hline \text{C: } (\forall x r(x)) \wedge (\neg(\forall x r(x))) \end{array}$$

6. Consider the following argument (domain is the set of all people):

   Every student pays tuition. Anyone who pays tuition can take courses. Sam is not allowed to take courses. Therefore, it is not true that anyone is a student.

   (a) Using the predicate symbols S(x) for *x is a student*, T(x) for *x pays tuition*, and C(x) for *x can take courses* state the argument as a theorem.

   (b) Provide a proof for the theorem.

7. For each of the following formulas, state if it is satisfiable or not. If satisfiable, give a model with natural numbers as the domain. If not, explain why it is not satisfiable.

   (a) $(\forall x(p(x) \rightarrow q(x)) \rightarrow (\forall x p(x) \rightarrow \forall x q(x))$

   (b) $(\forall x p(x) \rightarrow \forall x q(x)) \rightarrow (\forall x(p(x) \rightarrow q(x))$