1. Consider the following relational database schema:

   Consider the following relations of the gradebook database.

   STUDENTS(SID,fname,lname)
   COURSES(CNO,title)
   ENROLLS(SID,CNO)

   The key attributes are shown in all capital letters. The STUDENTS table records information about students, the COURSES table records information about courses and the ENROLLS table records information about which student is enrolled in which course.

   Write Datalog programs to answer the following queries.

   (a) Get the names of students who have enrolled in the course titled Automata.
   (b) Get the names of students who have enrolled in ONLY one course.
   (c) Get the names of students who have enrolled in EVERY course in which the student with SID = 1111 has enrolled.
   (d) Get the names of students who have enrolled in ONLY those courses in which the student with SID = 1111 has enrolled.
   (e) Get the names of students who have enrolled in EXACTLY THE SAME courses in which the student with SID = 1111 has enrolled.

2. Consider the following Datalog program.

   \[
   \begin{align*}
   P(X,Y) & : - Q(X,Y). \\
   P(X,Y) & : - Q(X,Z), P(Z,Y). \\
   Q(a,b). \\
   Q(b,c). \\
   Q(c,d). \\
   Q(d,e). \\
   \end{align*}
   \]

   (a) List the Herbrand Universe for the database.
   (b) How many elements does the Herbrand Base contain.
   (c) For each of the following Herbrand Interpretations, determine if it is a Herbrand Model for the Datalog program; If it is a Herbrand model, is it a minimal Herbrand model? Explain your answer.

   \[
   \begin{align*}
   I_1 & = \{ Q(a,b), Q(b,c), Q(c,d), Q(d,e), \\
           & \quad P(a,b), P(b,c), P(c,d), P(d,e) \} \\
   I_2 & = \{ Q(a,b), Q(b,c), Q(c,d), Q(d,e), \\
           & \quad P(a,b), P(b,c), P(c,d), P(d,e), P(a,c), P(b,d), P(c,e) \} \\
   I_3 & = \{ Q(a,b), Q(b,c), Q(c,d), Q(d,e), \\
           & \quad P(a,b), P(b,c), P(c,d), P(d,e), P(a,c), P(b,d), P(c,e), \\
           & \quad P(a,b), P(b,c), P(c,d), P(d,e), P(a,c), P(b,d), P(c,e), \\
           & \quad P(a,b), P(b,c), P(c,d), P(d,e), P(a,c), P(b,d), P(c,e), \\
           & \quad P(a,b), P(b,c), P(c,d), P(d,e), P(a,c), P(b,d), P(c,e), \\
           & \quad P(a,b), P(b,c), P(c,d), P(d,e), P(a,c), P(b,d), P(c,e), \\
           & \quad P(a,b), P(b,c), P(c,d), P(d,e), P(a,c), P(b,d), P(c,e), \\
           & \quad P(a,b), P(b,c), P(c,d), P(d,e), P(a,c), P(b,d), P(c,e), \\
           & \quad P(a,b), P(b,c), P(c,d), P(d,e), P(a,c), P(b,d), P(c,e), \\
           & \quad P(a,b), P(b,c), P(c,d), P(d,e), P(a,c), P(b,d), P(c,e), \\
           & \quad P(a,b), P(b,c), P(c,d), P(d,e), P(a,c), P(b,d), P(c,e), \\
           & \quad P(a,b), P(b,c), P(c,d), P(d,e), P(a,c), P(b,d), P(c,e), \\
           & \quad P(a,b), P(b,c), P(c,d), P(d,e), P(a,c), P(b,d), P(c,e), \\
           & \quad P(a,b), P(b,c), P(c,d), P(d,e), P(a,c), P(b,d), P(c,e), \\
   \end{align*}
   \]
\[ I_4 = \{ Q(a,b), Q(b,c), Q(c,d), Q(d,e), \\
\quad P(a,b), P(b,c), P(c,d), P(d,e), P(a,c), P(b,d), P(c,e), \\
\quad P(a,d), P(b,e), P(a,e) \} \]

3. Consider the following Deductive Database, DB.

\begin{verbatim}
nonstop(boston,london).
nonstop(london,frankfurt).
nonstop(frankfurt,newyork).
nonstop(newyork,boston).
air_route(Dep,Dest) :- nonstop(Dep,Dest), Dep <> Dest.
air_route(Dep,Dest) :- nonstop(Dep,Stop),
    air_route(Stop,Dest), Dep <> Dest.
\end{verbatim}

What is the value of \( T_{DB}(T_{DB}(I)) \) where \( I \) is the following Herbrand Interpretation:

\[ I = \{ \text{nonstop(boston,newyork), nonstop(newyork,frankfurt),} \\
\quad \text{nonstop(frankfurt,boston)} \} \]

4. Consider the following IDB:

\begin{verbatim}
path(X,Y) :- red(X,Y).
path(X,Y) :- path(X,U), blue(U,V), path(V,Y).
\end{verbatim}

and the EDB relations:

\[
\begin{array}{c|c|c|c|c|c}
\text{RED} & \text{BLUE} \\
\hline
a & b & b & c \\
\hline
a & d & b & d \\
\hline
c & d & b & e \\
\hline
d & f & & \\
\hline
e & f & & \\
\end{array}
\]

If we think of \textit{red} and \textit{blue} as representing red and blue arcs in a graph, then \textit{path} represents paths of alternating red and blue arcs, beginning and ending with a red arc.

(a) Write the DATALOG EQUATIONS for the IDB predicate \textit{path}.

(b) Write the INCREMENTAL DATALOG EQUATIONS for the IDB predicate \textit{path}.

(c) Apply the \textit{Semi-Naive Algorithm} to the above database. Show the values of all the variables after every iteration.

\[
\begin{array}{c|c|c}
\text{Iteration} & \text{PATH} & \text{DELTA-PATH} \\
\hline
1 & & \\
\end{array}
\]

5. Consider the following DATALOG program.
p(X,Y) :- b(X,Y).
p(X,Y) :- b(X,Z), p(Z,Y).
e(X,Y) :- g(X,Y), not p(X,Y).
a(X,Y) :- e(X,Y).
a(X,Y) :- e(X,Z), a(Z,Y).
b(1,2).
b(2,1).
b(3,4).
b(4,3).
g(2,3).
g(3,2).

(a) Determine the stratum of each predicate symbol.

(b) Construct the minimal model using the Stratified-Negation adaptation of the Naive algorithm (Show Datalog equations and values of relations after each iteration and stratum).

6. Consider the following database:

  murderer(X) :- ingarden(X), suspect(X).
  ingarden(X) :- not inhouse(X).
  ingarden(dale) :- not ingarden(peter).
  inhouse(jessica).
  suspect(jessica).
  suspect(dale).
  suspect(peter).

(a) List all the minimal models of the set of clauses. (Hint: move the negated atoms from body of rules to head and create disjunctive formulas).

(b) What atoms of the Herbrand base can be assumed to be false using CWA?