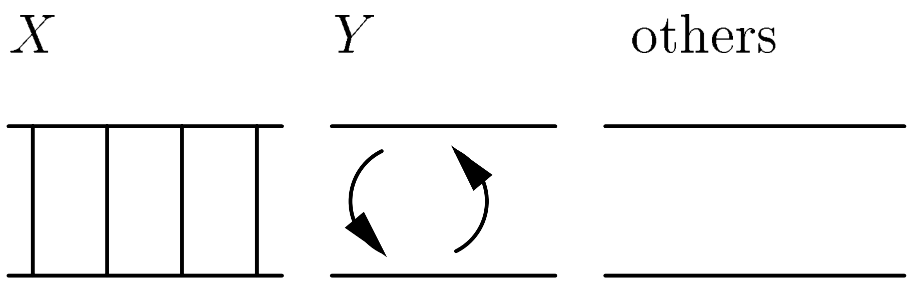
Multivalued Dependencies

The *multivalued dependency* $X \twoheadrightarrow Y$ holds in a relation R if whenever we have two tuples of R that agree in all the attributes of X, then we can swap their Y components and get two new tuples that are also in R.



Example

`Drinkers(name, addr, phones,beersLiked)` with MVD `name` $\twoheadrightarrow$ `phones`. If `Drinkers` has the two tuples:

name	addr	phones	beersLiked
sue	a	$p1$	$b1$
sue	a	$p2$	$b2$

it must also have the same tuples with `phones` components swapped:

name	addr	phones	beersLiked
sue	a	$p1$	$b2$
sue	a	$p2$	$b1$

- Note: we must check this condition for *all* pairs of tuples that agree on `name`, not just one pair.

MVD Rules

1. Every FD is an MVD.

   - Because if $X \to Y$, then swapping Y's between tuples that agree on X doesn't create new tuples.

   - Example, in `Drinkers`: `name` $\twoheadrightarrow$ `addr`.

2. *Complementation*: if $X \twoheadrightarrow Y$, then $X \twoheadrightarrow Z$, where Z is all attributes not in X or Y.

   - Example: since `name` $\twoheadrightarrow$ `phones` holds in `Drinkers`, so does `name` $\twoheadrightarrow$ `addr beersLiked`.

Splitting Doesn't Hold

Sometimes you need to have several attributes on the right of an MVD. For example:

`Drinkers(name, areaCode, phones, beersLiked, beerManf)`

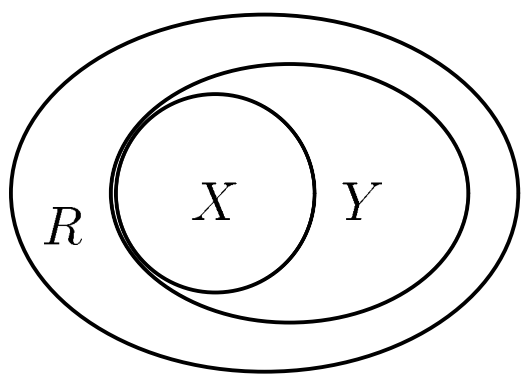
name	areaCode	phones	BeersLiked	beerManf
Sue	650	555-1111	Bud	A.B.
Sue	650	555-1111	WickedAle	Pete's
Sue	415	555-9999	Bud	A.B.
Sue	415	555-9999	WickedAle	Pete's

- `name` $\rightarrow\!\!\rightarrow$ `areaCode phones` holds, but neither `name` $\rightarrow\!\!\rightarrow$ `areaCode` nor `name` $\rightarrow\!\!\rightarrow$ `phones` do.

4NF

Eliminate redundancy due to multiplicative effect of MVD's.

- Roughly: treat MVD's as FD's for decomposition, but not for finding keys.
- Formally: R is in Fourth Normal Form if whenever MVD $X \twoheadrightarrow Y$ is *nontrivial* (Y is not a subset of X, and $X \cup Y$ is not all attributes), then X is a superkey.
  - Remember, $X \rightarrow Y$ implies $X \twoheadrightarrow Y$, so 4NF is more stringent than BCNF.
- Decompose R, using 4NF violation $X \twoheadrightarrow Y$, into XY and $X \cup (R - Y)$.



Example

`Drinkers(name, addr, phones, beersLiked)`

- FD: $\texttt{name} \rightarrow \texttt{addr}$
- Nontrivial MVD's: $\texttt{name} \twoheadrightarrow \texttt{phones}$ and $\texttt{name} \twoheadrightarrow \texttt{beersLiked}$.
- Only key: $\{\texttt{name, phones, beersLiked}\}$
- All three dependencies above violate 4NF.
- Successive decomposition yields 4NF relations:

  $\texttt{D1(}\underline{\texttt{name}}\texttt{, addr)}$
  $\texttt{D2(}\underline{\texttt{name}}\texttt{, }\underline{\texttt{phones}}\texttt{)}$
  $\texttt{D3(}\underline{\texttt{name}}\texttt{, }\underline{\texttt{beersLiked}}\texttt{)}$

Relational Algebra

A small set of operators that allow us to manipulate relations in limited but useful ways. The operators are:

1. Union, intersection, and difference: the usual set operators.
   - But the relation schemas must be the same.
2. *Selection*: Picking certain rows from a relation.
3. *Projection*: Picking certain columns.
4. *Products and joins*: Composing relations in useful ways.
5. *Renaming* of relations and their attributes.

Selection

$$R_1 = \sigma_C(R_2)$$

where C is a condition involving the attributes of relation R_2.

Example

Relation `Sells`:

bar	beer	price
Joe's	Bud	2.50
Joe's	Miller	2.75
Sue's	Bud	2.50
Sue's	Coors	3.00

`JoeMenu` = $\sigma_{bar=Joe's}$`(Sells)`

bar	beer	price
Joe's	Bud	2.50
Joe's	Miller	2.75

Projection

$$R_1 = \pi_L(R_2)$$

where L is a list of attributes from the schema of R_2.

Example

$\pi_{beer,price}$(`Sells`)

beer	price
Bud	2.50
Miller	2.75
Coors	3.00

- Notice elimination of duplicate tuples.

Product

$$R = R_1 \times R_2$$

pairs each tuple t_1 of R_1 with each tuple t_2 of R_2 and puts in R a tuple $t_1 t_2$.

Theta-Join

$$R = R_1 \underset{C}{\bowtie} R_2$$

is equivalent to $R = \sigma_C(R_1 \times R_2)$.

Example

`Sells` =

bar	beer	price
Joe's	Bud	2.50
Joe's	Miller	2.75
Sue's	Bud	2.50
Sue's	Coors	3.00

`Bars` =

name	addr
Joe's	Maple St.
Sue's	River Rd.

`BarInfo = Sells` $\underset{Sells.Bar=Bars.Name}{\bowtie}$ `Bars`

bar	beer	price	name	addr
Joe's	Bud	2.50	Joe's	Maple St.
Joe's	Miller	2.75	Joe's	Maple St.
Sue's	Bud	2.50	Sue's	River Rd.
Sue's	Coors	3.00	Sue's	River Rd.

Natural Join

$$R = R_1 \bowtie R_2$$

calls for the theta-join of R_1 and R_2 with the condition that all attributes of the same name be equated. Then, one column for each pair of equated attributes is projected out.

Example

Suppose the attribute `name` in relation `Bars` was changed to `bar`, to match the bar name in `Sells`.

`BarInfo = Sells` $\bowtie$ `Bars`

bar	beer	price	addr
Joe's	Bud	2.50	Maple St.
Joe's	Miller	2.75	Maple St.
Sue's	Bud	2.50	River Rd.
Sue's	Coors	3.00	River Rd.

Renaming

$\rho_{S(A_1,\ldots,A_n)}(R)$ produces a relation identical to R but named S and with attributes, in order, named $A_1, \ldots, A_n$.

Example

`Bars` =

name	addr
Joe's	Maple St.
Sue's	River Rd.

$\rho_{R(bar,addr)}(\texttt{Bars})$ =

bar	addr
Joe's	Maple St.
Sue's	River Rd.

- The name of the above relation is R.

Combining Operations

Algebra =

1. Basis arguments +
2. Ways of constructing expressions.

For relational algebra:

1. Arguments = variables standing for relations + finite, constant relations.
2. Expressions constructed by applying one of the operators + parentheses.

- Query = expression of relational algebra.

Operator Precedence

The normal way to group operators is:

1. Unary operators σ, π, and ρ have highest precedence.

2. Next highest are the "multiplicative" operators, $\bowtie$, $\underset{C}{\bowtie}$, and $\times$.

3. Lowest are the "additive" operators, $\cup$, $\cap$, and $-$.

- But there is no universal agreement, so we always put parentheses *around* the argument of a unary operator, and it is a good idea to group all binary operators with parentheses *enclosing* their arguments.

Example

Group $R \cup \sigma S \bowtie T$ as $R \cup (\sigma(S) \bowtie T)$.

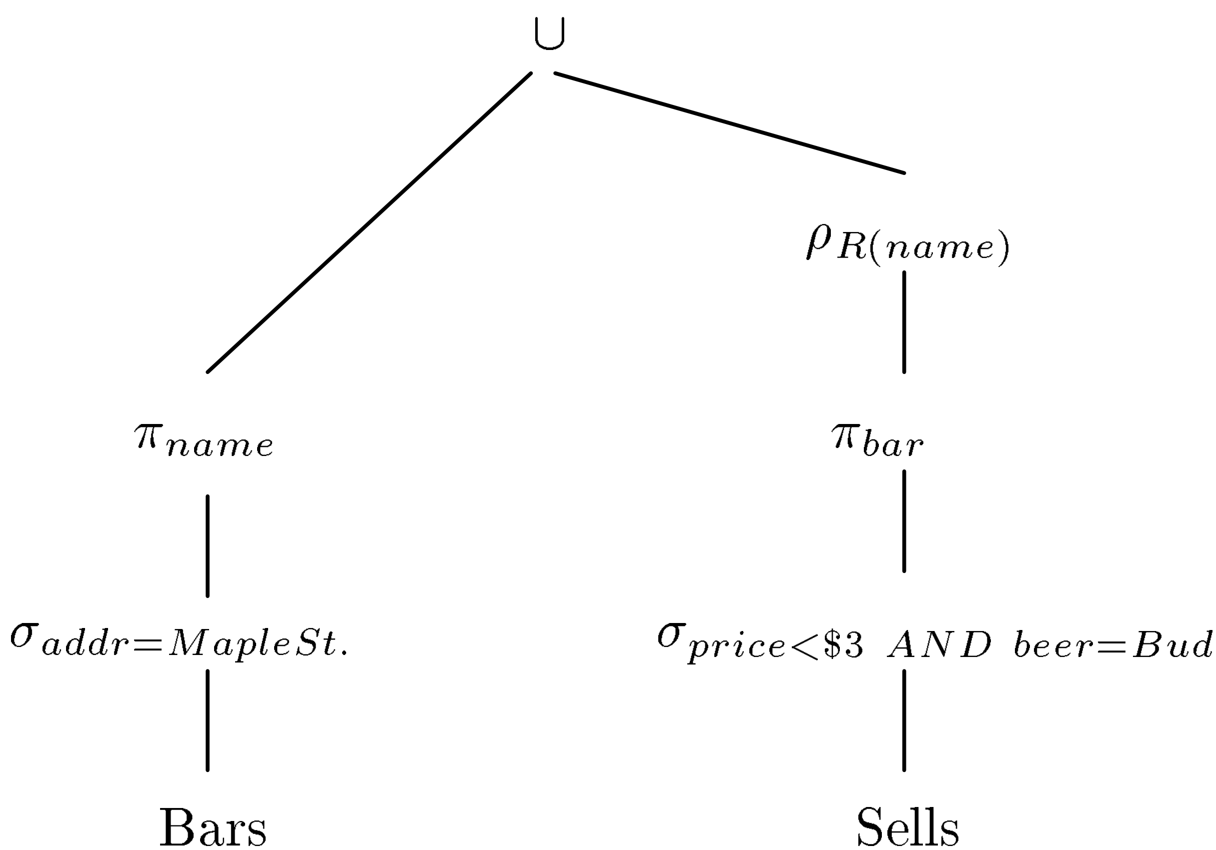
Each Expression Needs a Schema

- If $\cup$, $\cap$, $-$ applied, schemas are the same, so use this schema.
- Projection: use the attributes listed in the projection.
- Selection: no change in schema.
- Product $R \times S$: use attributes of R and S.
  - But if they share an attribute A, prefix it with the relation name, as $R.A$, $S.A$.
- Theta-join: same as product.
- Natural join: use attributes from each relation; common attributes are merged anyway.
- Renaming: whatever it says.

Example

Find the bars that are either on Maple Street or sell Bud for less than \$3.

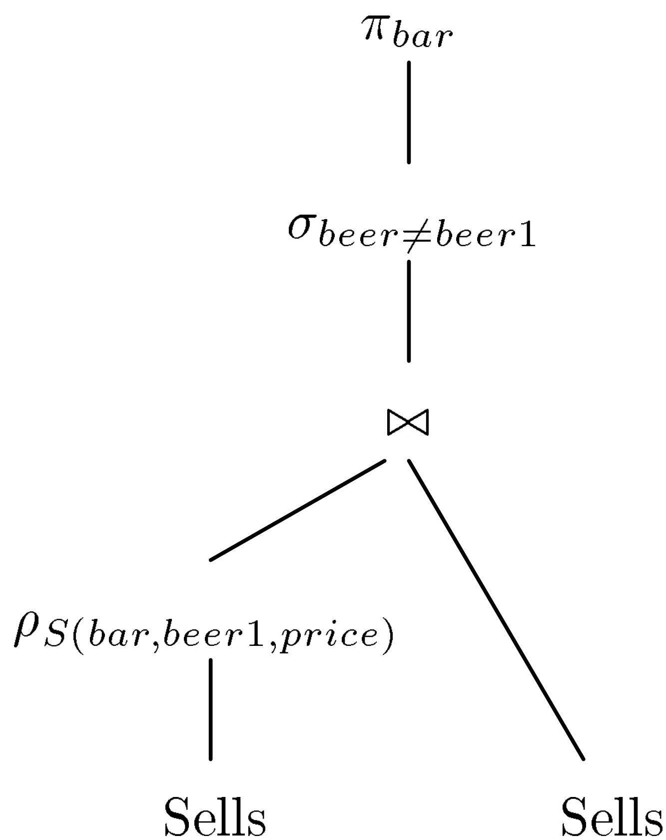
```
Sells(bar, beer, price)
Bars(name, addr)
```



Example

Find the bars that sell two different beers at the same price.

`Sells(bar, beer, price)`



Linear Notation for Expressions

- Invent new names for intermediate relations, and assign them values that are algebraic expressions.
- Renaming of attributes implicit in schema of new relation.

Example

Find the bars that are either on Maple Street or sell Bud for less than \$3.

```
Sells(bar, beer, price)
Bars(name, addr)
```

R1(name) := $\pi_{name}(\sigma_{addr=Maple\ St.}(\texttt{Bars}))$

R2(name) :=

$\pi_{bar}(\sigma_{beer=Bud\ AND\ price<\$3}(\texttt{Sells}))$

R3(name) := R1 $\cup$ R2

Why Decomposition "Works"?

What does it mean to "work"? Why can't we just tear sets of attributes apart as we like?

- Answer: the decomposed relations need to represent the same information as the original.
  - We must be able to reconstruct the original from the decomposed relations.

Projection and Join Connect the Original and Decomposed Relations

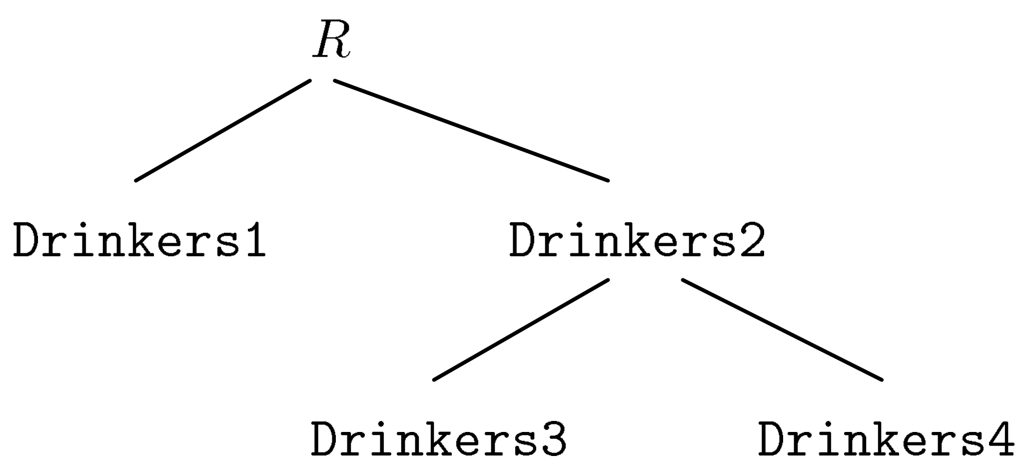
- Suppose R is decomposed into S and T. We project R onto S and onto T.

Example

$R =$

name	addr	beersLiked	manf	favoriteBeer
Janeway	Voyager	Bud	A.B.	WickedAle
Janeway	Voyager	WickedAle	Pete's	WickedAle
Spock	Enterprise	Bud	A.B.	Bud

- Recall we decomposed this relation as:



- Project onto `Drinkers1(name, addr, favoriteBeer)`:

name	addr	favoriteBeer
Janeway	Voyager	WickedAle
Spock	Enterprise	Bud

- Project onto `Drinkers3(beersLiked, manf)`:

beersLiked	manf
Bud	A.B.
WickedAle	Pete's

- Project onto `Drinkers4(name, beersLiked)`:

name	beersLiked
Janeway	Bud
Janeway	WickedAle
Spock	Bud

Reconstruction of Original

Can we figure out the original relation from the decomposed relations?

- Sometimes, if we natural join the relations.

Example

`Drinkers3` $\bowtie$ `Drinkers4` =

name	beersLiked	manf
Janeway	Bud	A.B.
Janeway	WickedAle	Pete's
Spock	Bud	A.B.

- Join of above with `Drinkers1` = original R.

Theorem

Suppose we decompose a relation with schema XYZ into XY and XZ and project the relation for XYZ onto XY and XZ. Then $XY \bowtie XZ$ is *guaranteed* to reconstruct XYZ if and only if $X \twoheadrightarrow Y$ (or equivalently, $X \twoheadrightarrow Z$.

- Usually, the MVD is really a FD, $X \rightarrow Y$ or $X \rightarrow Z$.
- BCNF: When we decompose XYZ into XY and XZ, it is because there is a FD $X \rightarrow Y$ or $X \rightarrow Z$ that violates BCNF.
  - ❖ Thus, we can always reconstruct XYZ from its projections onto XY and XZ.
- 4NF: when we decompose XYZ into XY and XZ, it is because there is an MVD $X \twoheadrightarrow Y$ or $X \twoheadrightarrow Z$ that violates 4NF.
  - ❖ Again, we can reconstruct XYZ from its projections onto XY and XZ.