Stratified Negation

- Negation wrapped inside a recursion makes no sense.
- Even when negation and recursion are separated, there can be ambiguity about what the rules mean, and some one meaning must be selected.
- *Stratified negation* is an additional restraint on recursive rules (like safety) that solves both problems:
  1. It rules out negation wrapped in recursion.
  2. When negation is separate from recursion, it yields the intuitively correct meaning of rules (the *stratified model*).

Problem with Recursive Negation

Consider:

```
P(x) <- Q(x) AND NOT P(x)
```

- $Q = \mathrm{EDB} = \{1, 2\}$.
- Compute IDB P iteratively?
  - Initially, $P = \emptyset$.
  - Round 1: $P = \{1, 2\}$.
  - Round 2: $P = \emptyset$, etc., etc.

Strata

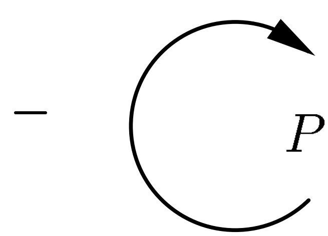
Intuitively: stratum of an IDB predicate = maximum number of negations you can pass through on the way to an EDB predicate.

- Must not be ∞ in "stratified" rules.
- Define *stratum graph*:
  - Nodes = IDB predicates.
  - Arc $P \rightarrow Q$ if Q appears in the body of a rule with head P.
  - Label that arc $-$ if Q is in a negated subgoal.

Example

Which target nodes cannot be reached from any source node?

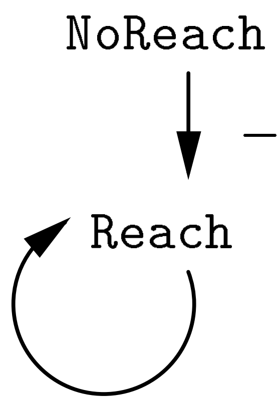
```
P(x) <- Q(x) AND NOT P(x)
```



Example

```
Reach(x) <- Source(x)
Reach(x) <- Reach(y) AND Arc(y,x)

NoReach(x) <- Target(x)
    AND NOT Reach(x)
```



Computing Strata

Stratum of an IDB predicate A = maximum number of $-$ arcs on any path from A in the stratum graph.

Examples

- For first example, stratum of P is ∞.
- For second example, stratum of `Reach` is 0; stratum of `NoReach` is 1.

Stratified Negation

A Datalog program with recursion and negation is *stratified* if every IDB predicate has a finite stratum.

Stratified Model

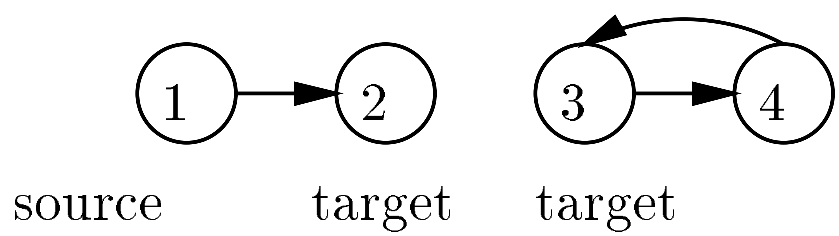
If a Datalog program is stratified, we can compute the relations for the IDB predicates lowest-stratum-first.

Example

```
Reach(x) <- Source(x)
Reach(x) <- Reach(y) AND Arc(y,x)

NoReach(x) <- Target(x)
    AND NOT Reach(x)
```

- EDB:
  - `Source` $= \{1\}$.
  - `Arc` $= \{(1,2),\ (3,4),\ (4,3)\}$.
  - `Target` $= \{2,3\}$.



- First compute `Reach` $= \{1,2\}$ (stratum 0).
- Next compute `NoReach` $= \{3\}$.

Is the Stratified Solution "Obvious"?

Not really.

- There is another model that makes the rules true no matter what values we substitute for the variables.
  - `Reach` $= \{1, 2, 3, 4\}$.
  - `NoReach` $= \emptyset$.
- Remember: the only way to make a Datalog rule false is to find values for the variables that make the body true and the head false.
  - For this model, the heads of the rules for `Reach` are true for all values, and in the rule for `NoReach` the subgoal `NOT Reach(x)` assures that the body cannot be true.

SQL3 Recursion

```
WITH
    stuff that looks like Datalog rules
an SQL query about EDB, IDB
```

- Rule =

  `[RECURSIVE]` R(<arguments>) `AS`
      SQL query

Example

Find Sally's cousins, using EDB `Par(child, parent)`.

```
WITH
    Sib(x,y) AS
        SELECT p1.child, p2,child
        FROM Par p1, Par p2
        WHERE p1.parent = p2.parent
            AND p1.child <> p2.child,

    RECURSIVE Cousin(x,y) AS
        Sib
            UNION
        (SELECT p1.child, p2.child
         FROM Par p1, Par p2, Cousin
         WHERE p1.parent = Cousin.x
             AND p2.parent = Cousin.y
        )

SELECT y
FROM Cousin
WHERE x = 'Sally';
```

Plan for Describing Legal SQL3 recursion

1. Define "monotonicity," a property that generalizes "stratification."

2. Generalize stratum graph to apply to SQL queries instead of Datalog rules.

   - (Non)monotonicity replaces `NOT` in subgoals.

3. Define semantically correct SQL3 recursions in terms of stratum graph.

Monotonicity

If relation P is a function of relation Q (and perhaps other things), we say P is *monotone* in Q if adding tuples to Q cannot cause any tuple of P to be deleted.

Monotonicity Example

In addition to certain negations, an aggregation can cause nonmonotonicity.

```
Sells(bar, beer, price)

SELECT AVG(price)
FROM Sells
WHERE bar = 'Joe''s Bar';
```

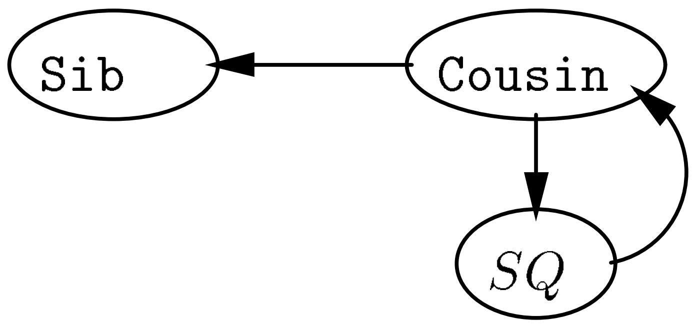
- Adding to `Sells` a tuple that gives a new beer Joe sells will usually change the average price of beer at Joe's.
- Thus, the former result, which might be a single tuple like (2.78) becomes another single tuple like (2.81), and the old tuple is lost.

Generalizing Stratum Graph to SQL

- Node for each relation defined by a "rule."
- Node for each subquery in the "body" of a rule.
- Arc $P \rightarrow Q$ if
  a) P is "head" of a rule, and Q is a relation appearing in the `FROM` list of the rule (not in the `FROM` list of a subquery), as argument of a `UNION`, etc.
  b) P is head of a rule, and Q is a subquery directly used in that rule (not nested within some larger subquery).
  c) P is a subquery, and Q is a relation or subquery used directly within P [analogous to (a) and (b) for rule heads].
- Label the arc $-$ if P is *not* monotone in Q.
- Requirement for legal SQL3 recursion: finite strata only.

Example

For the Sib/Cousin example, there are three nodes: `Sib`, `Cousin`, and SQ (the second term of the union in the rule for `Cousin`).



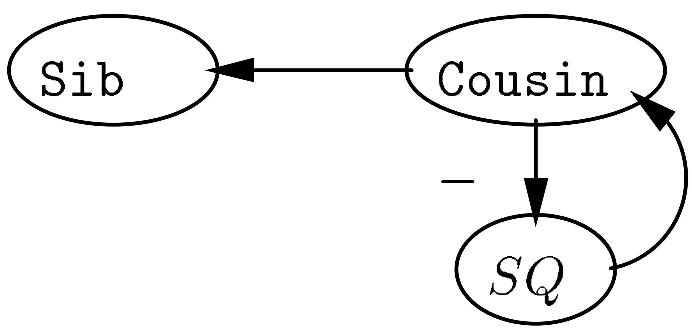
- No nonmonotonicity, hence legal.

A Nonmonotonic Example

Change the `UNION` to `EXCEPT` in the rule for `Cousin`.

```
RECURSIVE Cousin(x,y) AS
     Sib
          EXCEPT
     (SELECT p1.child, p2.child
      FROM Par p1, Par p2, Cousin
      WHERE p1.parent = Cousin.x
           AND p2.parent = Cousin.y
     )
```

- Now, adding to the result of the subquery can delete `Cousin` facts; i.e., `Cousin` is nonmonotone in SQ.



- Infinite number of $-$'s in cycle, so illegal in SQL3.

Another Example: `NOT` Doesn't Mean Nonmonotone

Leave `Cousin` as it was, but negate one of the conditions in the where-clause.

```
RECURSIVE Cousin(x,y) AS
    Sib
        UNION
    (SELECT p1.child, p2.child
     FROM Par p1, Par p2, Cousin
     WHERE p1.parent = Cousin.x
         AND NOT (p2.parent = Cousin.y)
    )
```

- You might think that SQ depends negatively on `Cousin`, but it doesn't.
  - ✧ If I add a new tuple to `Cousin`, all the old tuples still exist and yield whatever tuples in SQ they used to yield.
  - ✧ In addition, the new `Cousin` tuple might combine with old $p1$ and $p2$ tuples to yield something new.