c. A grade record refers to a student (Ssn), a particular section, and a grade (Grade).

Design a relational database schema for this database application. First show all the functional dependencies that should hold among the attributes. Then design relation schemas for the database that are each in 3NF or BCNF. Specify the key attributes of each relation. Note any unspecified requirements and make appropriate assumptions to render the specification complete.

Prove or disprove the following inference rules for functional dependencies. A proof can be made either by a proof argument or by using inference rules IR1 through IR3. A disproof should be performed by demonstrating a relation instance that satisfies the conditions and functional dependencies in the left-hand side of the inference rule but does not satisfy the dependencies in the right-hand side.

a. \([W \rightarrow Y, X \rightarrow Z] \models [WX \rightarrow Y]\)
b. \([X \rightarrow Y] \text{ and } Y \supseteq Z \models [X \rightarrow Z]\)
c. \([X \rightarrow Y, X \rightarrow W, WY \rightarrow Z] \models [X \rightarrow Z]\)
d. \([XY \rightarrow Z, Y \rightarrow W] \models [XW \rightarrow Z]\)
e. \([X \rightarrow Z, Y \rightarrow Z] \models [X \rightarrow Y]\)
f. \([X \rightarrow Y, XY \rightarrow Z] \models [X \rightarrow Z]\)
g. \([X \rightarrow Y, Z \rightarrow W] \models [XZ \rightarrow YW]\)
h. \([XY \rightarrow Z, Z \rightarrow X] \models [Z \rightarrow Y]\)
i. \([X \rightarrow Y, Y \rightarrow Z] \models [X \rightarrow YZ]\)
j. \([XY \rightarrow Z, Z \rightarrow W] \models [X \rightarrow W]\)

Consider the following two sets of functional dependencies: \(F = \{A \rightarrow C, AC \rightarrow D, E \rightarrow AD, E \rightarrow H\}\) and \(G = \{A \rightarrow CD, E \rightarrow AH\}\). Check whether they are equivalent.

Consider the relation schema EMP_DEPT in Figure 10.3(a) and the following set \(G\) of functional dependencies on EMP_DEPT: \(G = \{\text{Ssn} \rightarrow \{\text{Ename}, \text{Bdate}, \text{Address}, \text{Dnumber}\}, \text{Dnumber} \rightarrow \{\text{Dname}, \text{Dmgr_ssn}\}\}\). Calculate the closures \(\{\text{Ssn}\}^+\) and \(\{\text{Dnumber}\}^+\) with respect to \(G\).

Is the set of functional dependencies \(G\) in Exercise 10.20 minimal? If not, try to find a minimal set of functional dependencies that is equivalent to \(G\). Prove that your set is equivalent to \(G\).

What update anomalies occur in the EMP_PROJ and EMP_DEPT relations of Figures 10.3 and 10.4?

In what normal form is the LOTS relation schema in Figure 10.11(a) with respect to the restrictive interpretations of normal form that take only the primary key into account? Would it be in the same normal form if the general definitions of normal form were used?
Prove that any relation schema with two attributes is in BCNF.

Why do spurious tuples occur in the result of joining the EMP_PROJ1 and EMP_LOCS relations of Figure 10.5 (result shown in Figure 10.6)?

Consider the universal relation $R = \{A, B, C, D, E, F, G, H, I, J\}$ and the set of functional dependencies $F = \{ A \rightarrow C, A \rightarrow D, D \rightarrow E, B \rightarrow F, F \rightarrow G, H, D \rightarrow I, J \}$. What is the key for $R$? Decompose $R$ into 3NF relations.

Repeat Exercise 10.26 for the following different set of functional dependencies $G = \{ A \rightarrow C, B \rightarrow D, E \rightarrow F, A \rightarrow G, H, A \rightarrow I, H \rightarrow J \}$.

Consider the following relation:

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>b1</td>
<td>c1</td>
<td>#1</td>
</tr>
<tr>
<td>10</td>
<td>b2</td>
<td>c2</td>
<td>#2</td>
</tr>
<tr>
<td>11</td>
<td>b4</td>
<td>c1</td>
<td>#3</td>
</tr>
<tr>
<td>12</td>
<td>b3</td>
<td>c4</td>
<td>#4</td>
</tr>
<tr>
<td>13</td>
<td>b1</td>
<td>c1</td>
<td>#5</td>
</tr>
<tr>
<td>14</td>
<td>b3</td>
<td>c4</td>
<td>#6</td>
</tr>
</tbody>
</table>

- **A.** Given the previous extension (state), which of the following dependencies may hold in the above relation? If the dependency cannot hold, explain why by specifying the tuples that cause the violation.
  - i. $A \rightarrow B$
  - ii. $B \rightarrow C$
  - iii. $C \rightarrow B$
  - iv. $B \rightarrow A$
  - v. $C \rightarrow A$

- **B.** Does the above relation have a potential candidate key? If it does, what is it? If it does not, why not?

Consider a relation $R(A, B, C, D, E)$ with the following dependencies:

$AB \rightarrow C$,

$CD \rightarrow E$,

$DE \rightarrow B$

Is $AB$ a candidate key of this relation? If not, is $ABD$? Explain your answer.

Consider the relation $R$, which has attributes that hold schedules of courses and sections at a university; $R = \{Course_no, Sec_no, Offering_dept, Credit_hours, Course_level, Instructor_ssn, Semester, Year, Days_hours, Room_no, No_of_students\}$. Suppose that the following functional dependencies hold on $R$:

$Course_no \rightarrow \{Offering_dept, Credit_hours, Course_level\}$

$Course_no, Sec_no, Semester, Year \rightarrow \{Days_hours, Room_no, No_of_students, Instructor_ssn\}$

$Room_no, Days_hours, Semester, Year \rightarrow \{Instructor_ssn, Course_no, Sec_no\}$

Try to determine which sets of attributes form keys of $R$. How would you normalize this relation?
Write each of the following dependencies as an FD:

a. The manufacturer and serial number uniquely identifies the drive.
b. A model number is registered by a manufacturer and therefore can't be used by another manufacturer.
c. All disk drives in a particular batch are the same model.
d. All disk drives of a certain model of a particular manufacturer have exactly the same capacity.

Show that $AB \rightarrow D$ is in the closure of 

$$\{AB \rightarrow C, CE \rightarrow D, A \rightarrow E\}$$

10.36. Consider the following relation:

$$R (\text{Doctor\#}, \text{Patient\#}, \text{Date}, \text{Diagnosis}, \text{Treat\_code}, \text{Charge})$$

In the above relation, a tuple describes a visit of a patient to a doctor along with a treatment code and daily charge. Assume that diagnosis is determined (uniquely) for each patient by a doctor. Assume that each treatment code has a fixed charge (regardless of patient). Is this relation in 2NF? Justify your answer and decompose if necessary. Then argue whether further normalization to 3NF is necessary, and if so, perform it.

10.37. Consider the following relation:

$$\text{CAR\_SALE (Car\_id, Option\_type, Option\_listprice, Sale\_date, Option\_discountedprice)}$$

This relation refers to options installed in cars (e.g., cruise control) that were sold at a dealership, and the list and discounted prices of the options.

If $\text{Car\_id} \rightarrow \text{Sale\_date}$ and $\text{Option\_type} \rightarrow \text{Option\_listprice}$
and $\text{Car\_id}, \text{Option\_type} \rightarrow \text{Option\_discountedprice}$,
argue using the generalized definition of the 3NF that this relation is not in 3NF. Then argue from your knowledge of 2NF, why it is not even in 2NF.

10.38. Consider a decomposed version of the above relation:

$$\text{ACTUAL\_OPTION\_PRICING (Car\_id, Option\_type, Option\_discountedprice)}$$

$$\text{CAR (Car\_id, Sale\_date)}$$

$$\text{OPTION (Option\_type, Option\_list\_price)}$$

Using the algorithm for lossless decomposition checking (Algorithm 11.1),
determine if this decomposition is indeed lossless.
11.22. Consider the example of normalizing the \textsc{Lots} relation in Section 10.4. Determine whether the decomposition of \textsc{Lots} into \{\textsc{Lots1AX}, \textsc{Lots1AY}, \textsc{Lots1B}, \textsc{Lots2}\} has the lossless join property, by applying Algorithm 11.1 and also by using the test under Property NJB.

11.23. Show how the MVDS \textsc{Ename} \to \textsc{Pname} and \textsc{Ename} \to \textsc{Dname} in Figure 11.4(a) may arise during normalization into 1NF of a relation, where the attributes \textsc{Pname} and \textsc{Dname} are multivalued.

11.24. Apply Algorithm 11.4(a) to the relation in Exercise 10.26 to determine a key for \textsc{R}. Create a minimal set of dependencies \textsc{G} that is equivalent to \textsc{F}, and apply the synthesis algorithm (Algorithm 11.4) to decompose \textsc{R} into 3NF relations.

11.25. Repeat Exercise 11.24 for the functional dependencies in Exercise 10.27.

11.26. Apply the decomposition algorithm (Algorithm 11.3) to the relation \textsc{R} and the set of dependencies \textsc{F} in Exercise 10.26. Repeat for the dependencies \textsc{G} in Exercise 10.27.

11.27. Apply Algorithm 11.4(a) to the relations in Exercises 10.29 and 10.30 to determine a key for \textsc{R}. Apply the synthesis algorithm (Algorithm 11.4) to decompose \textsc{R} into 3NF relations and the decomposition algorithm (Algorithm 11.3) to decompose \textsc{R} into BCNF relations.

11.28. Write programs that implement Algorithms 11.3 and 11.4.

11.29. Consider the following decompositions for the relation schema \textsc{R} of Exercise 10.26. Determine whether each decomposition has (1) the dependency preservation property, and (2) the lossless join property, with respect to \textsc{F}. Also determine which normal form each relation in the decomposition is in.

a. \[D_1 = \{R_1, R_2, R_3, R_4, R_5\}; R_1 = \{A, B, C\}, R_2 = \{A, D, E\}, R_3 = \{B, F\}, R_4 = \{F, G, H\}, R_5 = \{D, I, J\}\]

b. \[D_2 = \{R_1, R_2, R_3\}; R_1 = \{A, B, C, D, E\}, R_2 = \{B, F, G, H\}, R_3 = \{D, I, J\} \]

c. \[D_3 = \{R_1, R_2, R_3, R_4, R_5\}; R_1 = \{A, B, C, D\}, R_2 = \{D, E\}, R_3 = \{B, F\}, R_4 = \{F, G, H\}, R_5 = \{D, I, J\}\]

11.30. Consider the relation \textsc{Refrig}(Model#, Year, Price, Manuf_plant, Color), which is abbreviated as \textsc{Refrig}(\textsc{M}, \textsc{Y}, \textsc{P}, \textsc{MP}, \textsc{C}), and the following set \textsc{F} of functional dependencies: \[\textsc{F} = \{\textsc{M} \to \textsc{MP}, \{\textsc{M}, \textsc{Y}\} \to \textsc{P}, \textsc{MP} \to \textsc{C}\}\]

a. Evaluate each of the following as a candidate key for \textsc{Refrig}, giving reasons why it can or cannot be a key: \{\textsc{M}\}, \{\textsc{M}, \textsc{Y}\}, \{\textsc{M}, \textsc{C}\}.

b. Based on the above key determination, state whether the relation \textsc{Refrig} is in 3NF and in BCNF, giving proper reasons.

c. Consider the decomposition of \textsc{Refrig} into \[D = \{R_1(\textsc{M}, \textsc{Y}, \textsc{P}), R_2(\textsc{M}, \textsc{MP}, \textsc{C})\}\]. Is this decomposition lossless? Show why. (You may consult the test under Property LJ1 in Section 11.1.4.)