#### Relational Algebra

### Set-theoretic operations:

attributes and the domains of the corresponding attributes in the two relations are the same Two relations are union-compatible if they have the same number of

Consider two relations r(R) and s(S) that are union-compatible (normally  ${\sf R}={\sf S}$ ).

Union:  $r \cup s = \{t | t \in r \text{ or } t \in s\}.$ 

Difference:  $r - s = \{t | t \in r \text{ and } t \notin s\}$ 

Intersection:  $r \cap s = \{t | t \in r \text{ and } t \in s\}$ 

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Cartesian Product: r(R) and s(S) on any schemes R and S.

$$r \times s = \{t_1.t_2 | t_1 \in r \text{ and } t_2 \in s\},$$

where, t1.t2 is the concatenation of tuples  $t_1$  and  $t_2$  to form a larger tuple.

### Example: set operations

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 $r \times s$ 

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## Relation-theoretic operations

Consider r(R) and s(S), two relations, where  $R=(A_1,...,A_n)$  and  $S=(B_1,...,B_m)$ 

Rename:  $r(C_1,...,C_n) = \{t | t \in r\}$  with schema  $(C_1,...,C_n)$ .

**Select:**  $\sigma_F(r) = \{t | t \in r \text{ and } t \text{ satisfies } F\}.$ 

examples how F is constructed) where F is a selection criteria involving constants and attributes of r. (will discuss in

 $\begin{aligned} \mathbf{Project:} \qquad & \pi_{D_1,...,D_p}(r) = \{t[D_1,...,D_p] | t \in r\} \\ & \text{where } D_i \text{ is one of } A_1,...,A_n. \end{aligned}$ 

**theta-Join:**  $r \bowtie_F s = \{t | (\exists u \in r) (\exists v \in s) (t = u.v \text{ and } F \text{ is satisfied by } u \text{ and } v) \}$ where F is a conjunction of formulas relating attributes of r with attributes of s. (will discuss in examples how F is constructed)

Natural Join:  $r \bowtie s = \{t | (\exists u \in r) (\exists v \in s) (t[R] = u \text{ and } t[S] = v)\}$ 

**Division:** Assume  $B_1,...,B_m\subset A_1,...,A_n$ .

$$r \div s = \{t | (\forall u \in s)(t.u \in r)\}$$

## Examples: relation-theoretic operations

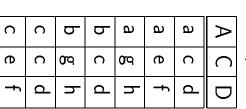
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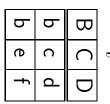
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$$\sigma_{A='b'}$$
 or  $_{C='c'}(r)$ 

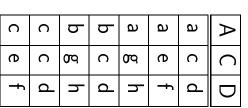
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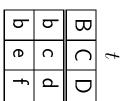
$$\pi_A(r)$$

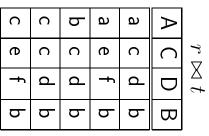


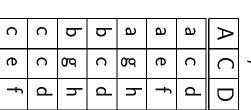


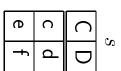
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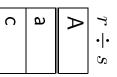












## Basic Relational Algebra Operations

- Basic set: union, difference, Cartesian product, rename, select, and project.
- none of them can be expressed in terms of the others.
- Intersection, theta-join, natural join, and division can be expressed in terms of the basic operators as follows:

Intersection:  $r \cap s = r - (r - s)$ theta Join:  $r \bowtie_F s = \sigma_F(r \times s)$ 

Natural Join:  $r \bowtie s = \pi_{R \cap S}(\sigma_F(r \times s))$ 

the common attributes of r and s are equal. where F is a selection condition which indicates that the tuple values under

**Division:**  $r \div s = \pi_{R-S}(r) - \pi_{R-S}((\pi_{R-S}(r) \times s) - r)$ 

equalities for simplicity. Even though relation schemes are defined as sequences, they are treated as sets in these

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# An explanation for the equality for division is in order!

ullet First, all candidate tuples for the result are calculated by the expression

$$\pi_{R-S}(r)$$

 $\bullet$  Next, these candidate tuples are combined with all tuples of s in the following expression

$$\pi_{R-S}(r) \times s$$

to give a relation containing all combinations of candidate tuples with all tuples of s.

• Since we are looking for tuples under the scheme R-S which combine with all tuples of s and are also present in r, if we subtract r from the previous expression, we will get all the combinations of tuples that are "missing" in r.

$$(\pi_{R-S}(r) \times s) - r$$

 $\bullet$  By projecting these tuples on R-S, we get all those tuples that should not go to the result in the following expression.

$$\pi_{R-S}((\pi_{R-S}(r)\times s)-r)$$

• Finally, we subtract this set from the set of all candidate tuples and obtain the output relation of the division operator.

$$r \div s = \pi_{R-S}(r) - \pi_{R-S}((\pi_{R-S}(r) \times s) - r)$$