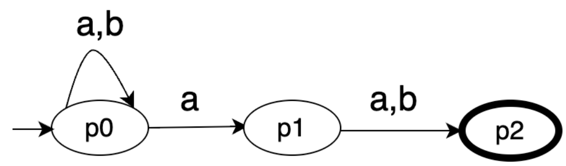
CSC 4510/6510 Automata
Exam 1 (Thursday, October 10, 2024)

1) Write a regular expression and construct a DFA for the language:

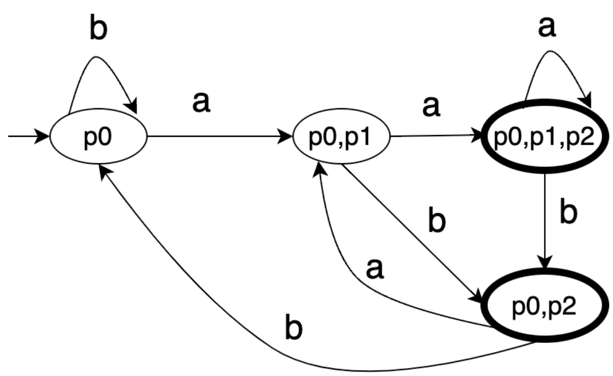
$L = \{ w \mid w \in \{a,b\}^*$ and w has an 'a' in the second from last position $\}$

$(a+b)^*a(a+b)$

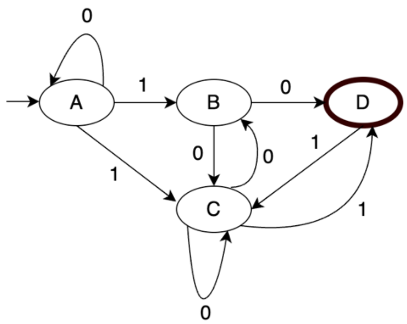
NFA

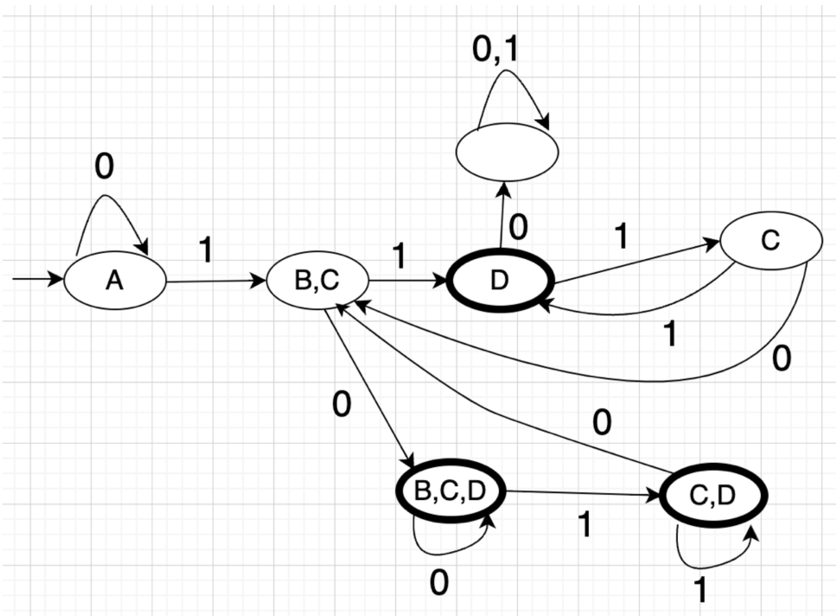


DFA

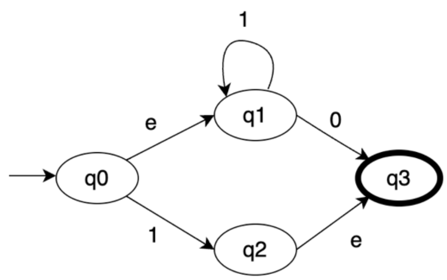


2) Convert the following NFA to a DFA using the NTD algorithm:





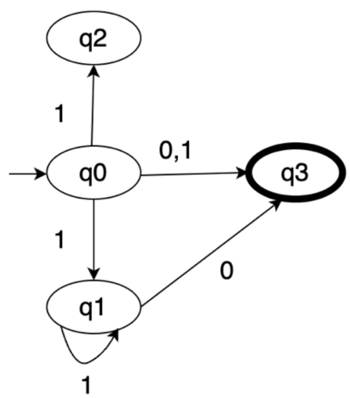
3) Apply the RMET algorithm to convert the following NFA with ε transitions to an NFA with no ε transitions:



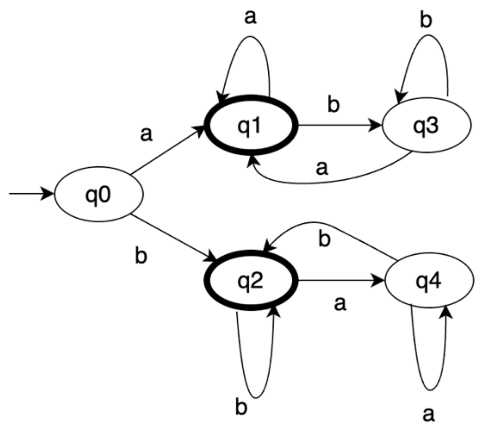
NFA without e-transitions

state	**e-closure**
q0	q0, q1
q1	q1
q2	q2, q3
q3	q3

	0	**1**
q0	q3	q1, q2, q3
q1	q3	q1
q2		
q3		



4) Using Arden's Lemma, derive the regular expression for the following DFA:



Initial Equations:

A0 = aA1 + bA2
A1 = aA1 + bA3 + e
A2 = aA4 + bA2 + e
A3 = aA1 + bA3
A4 = aA4 + bA2

Using Arden's Lemma on A3's equation, we get:

A3 = b*aA1

Using Arden's Lemma on A4's equation, we get:

A4 = a*bA2

Substituting these values in A1's and A2's equations, we get:

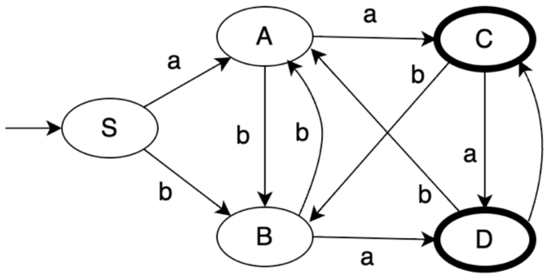
A1 = aA1 + bb*aA1 + e = (a + bb*a)A1 + e = (a+bb*a)*

A2 = bA2 + aa*bA1 + e = (b + aa*b)A2 + e = (b+aa*b)*

Substituting these values in A0's equations, we get:

A0 = a(a+bb*a)* + b(b+aa*b)*

5) Minimize the following DFA



A	Ø			
B	Ø			
C	X	X	X	
D	X	X	X	
	S	A	B	**C**

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Just marked (A,S) due to (A,C)
Just marked (B,S) due to (A,D)
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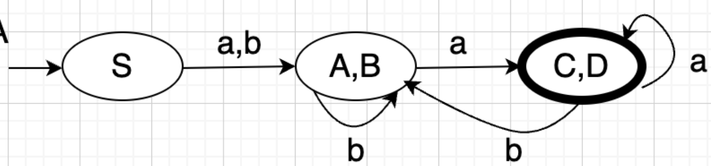
Indistinguishable pairs: (A,B), (C,D)

Equivalence classes of states: {S}, {A, B}, {C,D}

Minimized DFA



6) Precisely state the Pumping Lemma for regular languages.

For every regular language L, **there is a constant natural number** N such that **for every w in L**, with |w| >= N, **there exist words** u, v, x, such that w = uvx with |uv| <= N, |v| >= 1, and **for all natural numbers** i >=0, uv^ix is in L.

7) Using the Pumping Lemma for regular languages, prove that the language of palindromes described below is not regular:

PAL = { w | w ∈ $\{a,b\}^*$ and w = w^R }

Let PAL be regular.

Pumping Lemma guarantees us a constant N.

Choose w = a^Nba^N

Clearly, w is in PAL and |w| >= N.

Pumping Lemma says that w = uvx, with |uv| <= N, |v| >= 1, and for all i >=0, uv^ix is in PAL.

So, u = a^{k1} , v = a^{k2}, and x = $a^{k3}ba^N$, where k1 + k2 + k3 = N, k2 >= 1.

Choose i = 2.

So, a^{k1} a^{2k2} $a^{k3}ba^N$ is in PAL. i.e., $a^{N+k2}ba^N$ is in PAL. But this is a contradiction because number of a's before the b is not equal to number of a's after the b (since k2 >= 1), which makes $a^{N+k2}ba^N$ non-palindrome.

Therefore, PAL is not regular.

8) True or False. Explain your answer:

a) Every finite subset of a non-regular language is regular. **TRUE**.

Because every finite set is regular.

b) Every subset of a regular language is regular. **FALSE**.

Because $\{a^n b^n \mid n >= 0\}$ is not regular and it is a subset of L(a*b*) which is regular.

c) The set of words over alphabet { a, b } with equal number of a's and b's is regular. **FALSE**.

EQUAL $\cap$ L(a*b*) = $\{a^n b^n \mid n >= 0\}$

If EQUAL were regular, by closure under intersection, $\{a^n b^n \mid n >= 0\}$ would be regular, a contradiction. So, EQUAL must not be regular.

d) The regular expressions $(a^*b + bba)^*ba$ and $(a^*bba + bbaba)^*$ are equivalent. **FALSE**.

Because empty string belongs in $L((a^*b + bba)^*ba)$ but does not belong in $L((a^*bba + bbaba)^*)$.