Formal languages

01/03/96

— set of strings

ch. 1 Alphabets, Languages

1) Alphabet :- finite non-empty set of symbols

$$\Sigma_1 = \{a, b\}$$

$$\Sigma_2 = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

$$\Sigma_3 = \{a, b, c, \ldots\ldots, z\}$$

$$\Sigma_4 = \{\triangle, \square\}$$

2) **String/word** over alphabet Σ : finite sequence of symbols from Σ

3) **Language** over alphabet Σ is a set of strings over Σ

$$L_1 = \{ab, aab, aaab, \ldots\ldots\}$$

$$= \{w / w \text{ is a string over } \Sigma_1 \text{ and } w \text{ has } \underline{1} \text{ or more } a\text{'s followed by one}$$

4) **Star Closure of Σ**

Σ^* = set of all possible strings over Σ
= Universal Language

Operations on strings

1) **Length of string** = # of symbols in string

eg: $|abaa| = 4$

$|\epsilon| = 0$

2) **Concatenation of two strings**

Appending of one string to another is Concatenatio
Appending z to w is denoted by $w.z$ or wz

$w\epsilon = \epsilon w = w$
$|wz| = |w| + |z|$

3) **Exponentication**

$$w^n = \begin{cases} \epsilon, & n = 0 \\ ww^{n-1}, & n > 0 \end{cases}$$

$w^3 = www$
$w^0 = \epsilon$
$w = aab$
$w^3 = aabaabaab$

4) **Suffix, prefix**

x is a Suffix of w if there exists y such that $w=$
x is a Prefix of w if there exists y such that $w=$
y could be ϵ
if $y \neq \epsilon$ then proper Suffix/Prefix

5) Substring/Subword

x is a substring of w if there exists y, z such that $w = y\,x\,z$

6) Reverse

$$w^R = \begin{cases} w & \text{if } w = \epsilon \\ y^R a & \text{if } w = ay \ ; \ a \in \Sigma, y \in \Sigma^* \end{cases}$$

$$(xy)^R = y^R x^R$$

$$(x^R)^R = x$$

Operations on languages 01/06/9

Let L_1, L_2 be two languages

$$L_1 \cdot L_2 = \left\{ w_1 \cdot w_2 \;\middle|\; \begin{array}{l} w_1 \in L_1 \text{ and} \\ w_2 \in L_2 \end{array} \right\}$$

ex: $L_1 = \{a, bb\}$

$L_2 = \{aa, b\}$

$L_1 \cdot L_2 = \{aaa, ab, bbaa, bbb\}$

notation $L = L^1$

$L \cdot L = L^2$

$L \cdot L \cdot L = L^3$

Set operators

$L_1 \cup L_2 = \{w \mid w \in L_1 \text{ or } w \in L_2\}$ (union)

$L_1 \cap L_2 = \{w \mid w \in L_1 \text{ and } w \in L_2\}$ (intersectio

$L_1 - L_2 = \{w \mid w \in L_1 \text{ and } w \notin L_2\}$ (difference)

$\overline{L_1} = \Sigma^* - L_1$ (complement

Star - Closure

$$L^* = \bigcup_{i=0}^{\infty} L^i = L^0 \cup L^1 \cup L^2 \cup L^3 \cup L^4 \ldots\ldots\ldots$$

$$\Sigma = \{a, b\} \quad ; \quad L = \{a, bb\}$$

$$L^* = \underbrace{\{\epsilon\}}_{L^0} \cup \underbrace{\{a, bb\}}_{L^1} \cup \underbrace{\{aa, abb, bba, bbbb\}}_{L^2} \cup$$

$$\underbrace{\{aaa, aabb, abba, abbbb, bbaa, bbabb, bbbba, bbbbbb\}}_{L^3}$$

$$L^+ = \bigcup_{i=1}^{\infty} L^i$$

$$\ldots\ldots\ldots\ldots\{ \}_{L^{10}}$$

Summary of operations on languages

1) Union
2) Concatenation
3) Star closure
4) intersection
5) difference
6) Complement
7) Plus closure
8) exponentiation
9) reverse

1) $A^+ = A \cdot A^* = A^* \cdot A$

2) $(A^*)^* = A^*$

3) $(A^+)^+ = A^+$

Thm. $(A \cdot B)^R = B^R \cdot A^R$

$A = \{ab, b\} \qquad A^R = \{ba, b\}$

$B = \{aab, ba\} \qquad B^R = \{baa, ab\}$

$A \cdot B = \{ab\,aab, bba, \ldots\ldots \}$

$A \cdot B^R = \{ba\,baa, bab, \ldots\ldots \}$