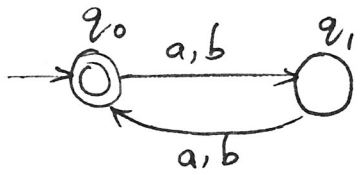


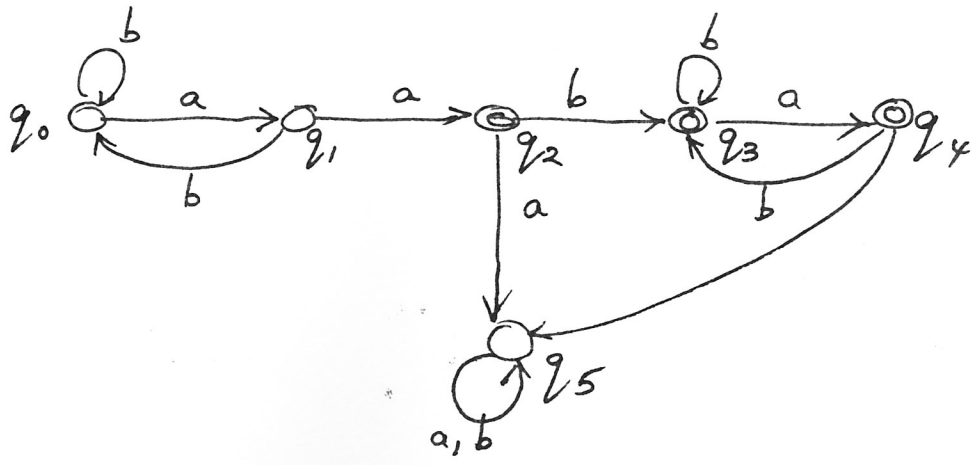
ex:



$$A_0 = aA_1 \cup bA_1 \cup \epsilon = (a \cup b)A_1 \cup \epsilon = \overbrace{(a \cup b)^A}^A A_0 \cup \epsilon$$

$$A_1 = (a \cup b)A_0$$

ex: words with one occurrence of 'aa'



$$\begin{aligned}
 A_0 &= bA_0 \cup aA_1 \\
 A_1 &= bA_0 \cup aA_2 \\
 A_2 &= bA_3 \cup aA_5 \cup \epsilon \\
 A_3 &= bA_3 \cup aA_4 \cup \epsilon \\
 A_4 &= bA_3 \cup aA_5 \cup \epsilon \\
 A_5 &= (a \cup b)A_5 = \phi \\
 &= (a \cup b)^+ A_5 \cup \phi \\
 A_5 &= (a \cup b)^+ \cdot \phi = \phi
 \end{aligned}$$

$$A_0 = bA_0 + aA_1$$

$$A_1 = bA_0 + aA_2$$

$$A_2 = bA_3 + \epsilon$$

$$A_3 = bA_3 + aA_4 + \epsilon$$

$$A_4 = bA_3 + \epsilon$$

$$A_3 = bA_3 + abA_3 + a + \epsilon$$
$$= (b+ab)A_3 + (a+\epsilon)$$

$$A_3 = (b+ab)^* (a+\epsilon)$$

$$A_2 = b(b+ab)^* (a+\epsilon) + \epsilon$$

$$A_1 = bA_0 + a[b(b+ab)^* (a+\epsilon) + \epsilon]$$

$$A_0 = bA_0 + abA_0 + aab(b+ab)^* (a+\epsilon) + \epsilon$$
$$= (b+ab)A_0 + aab(b+ab)^* (a+\epsilon) + \epsilon$$

$$A_0 = (b+ab)^* aab(b+ab)^* (a+\epsilon) + \epsilon$$

Arden's Lemma

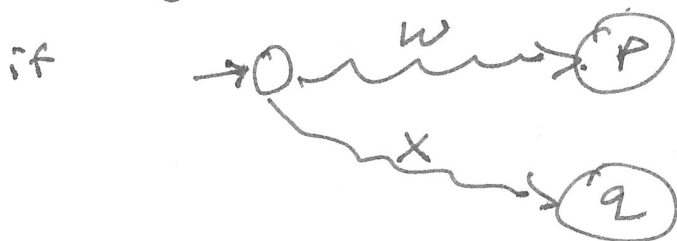
$$X = AX + B, \epsilon \notin A$$

then

$$X = A^* B$$

MIN

def distinguishable pair of states (p, q)



$$w \neq x$$

Alg (intuition)

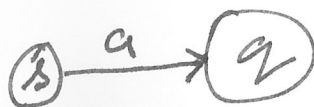
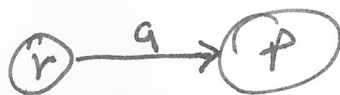
1. mark (p, q) as distinguishable if

$(p \in F \text{ and } q \notin F)$ or

$(p \notin F \text{ and } q \in F)$

2. take a pair (p, q) that is distinguishable

if (r, s)
exists s.t.

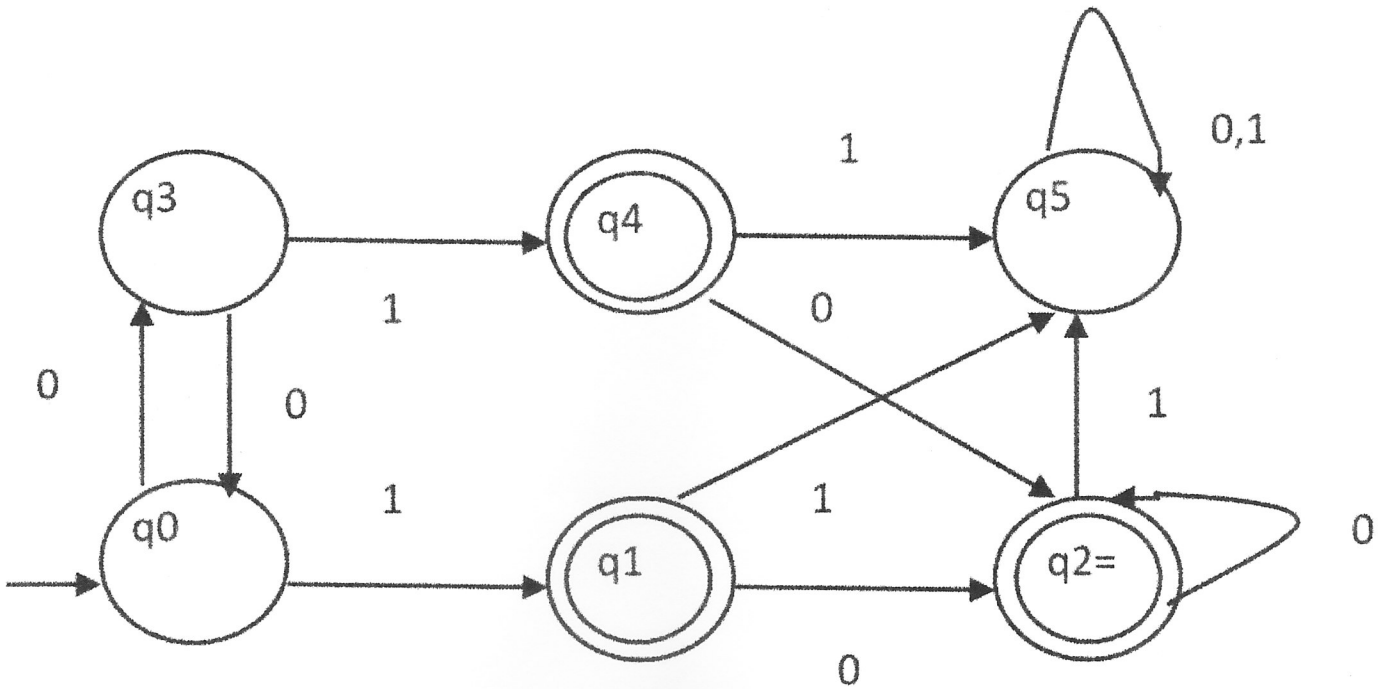


$$\neq$$

$$r \neq s$$

then (r, s) is distinguishable

~~d2-non-minimal fsm~~



| | | | | | |
|----|----|----|----|----|----|
| q1 | X | | | | |
| q2 | X | • | | | |
| q3 | • | X | X | | |
| q4 | X | • | • | X | |
| q5 | X | X | X | X | X |
| | q0 | q1 | q2 | q3 | q4 |

indistinguishable pairs

$$\left\{ (q_1, q_2), (q_2, q_4), (q_1, q_4), (q_0, q_3) \right\}$$

Equivalence classes = $\{q_0, q_3\}, \{q_1, q_2, q_4\}$

