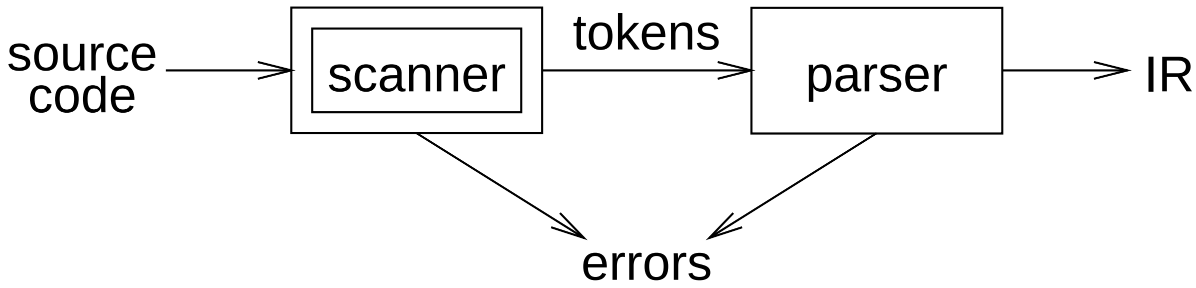
Chapter 2: Lexical Analysis

Scanner



- maps characters into *tokens* – the basic unit of syntax
  `x = x + y;`
  becomes
  $\langle$id, `x`$\rangle$ `=` $\langle$id, `x`$\rangle$ `+` $\langle$id, `y`$\rangle$ `;`
- character string value for a *token* is a *lexeme*
- typical tokens: *number*, *id*, `+`, `-`, `*`, `/`, `do`, `end`
- eliminates white space (*tabs, blanks, comments*)
- a key issue is speed
  $\Rightarrow$ use specialized recognizer (as opposed to `lex`)

Specifying patterns

A scanner must recognize various parts of the language's syntax

Some parts are easy:

white space

$$
\begin{array}{lcl}
\langle \text{ws} \rangle & ::= & \langle \text{ws} \rangle \texttt{' '} \\
 & | & \langle \text{ws} \rangle \texttt{'\textbackslash t'} \\
 & | & \texttt{' '} \\
 & | & \texttt{'\textbackslash t'}
\end{array}
$$

keywords and operators

specified as literal patterns: `do`, `end`

comments

opening and closing delimiters: `/* ··· */`

Specifying patterns

A scanner must recognize various parts of the language's syntax

Other parts are much harder:

identifiers

alphabetic followed by k alphanumerics (_, $, &, . . .)

numbers

integers: 0 or digit from 1-9 followed by digits from 0-9

decimals: integer `'.'` digits from 0-9

reals: (integer or decimal) `'E'` (+ or -) digits from 0-9

complex: `'('` real `','` real `')'`

We need a powerful notation to specify these patterns

Operations on languages

Operation	Definition
union of L and M written $L \cup M$	$L \cup M = \{s \mid s \in L \text{ or } s \in M\}$
concatenation of L and M written *LM*	$LM = \{st \mid s \in L \text{ and } t \in M\}$
Kleene closure of L written L^*	$L^* = \bigcup_{i=0}^{\infty} L^i$
positive closure of L written L^+	$L^+ = \bigcup_{i=1}^{\infty} L^i$

Regular expressions

Patterns are often specified as *regular languages*

Notations used to describe a regular language (or a regular set) include both *regular expressions* and *regular grammars*

Regular expressions (*over an alphabet* Σ):

1. ε is a RE denoting the set $\{\varepsilon\}$
2. if $a \in \Sigma$, then a is a RE denoting $\{a\}$
3. if r and s are REs, denoting $L(r)$ and $L(s)$, then:

    (r) is a RE denoting $L(r)$

    $(r) \mid (s)$ is a RE denoting $L(r) \bigcup L(s)$

    $(r)(s)$ is a RE denoting $L(r)L(s)$

    $(r)^*$ is a RE denoting $L(r)^*$

If we adopt a *precedence* for operators, the extra parentheses can go away. We assume *closure*, then *concatenation*, then *alternation* as the order of precedence.

Examples

identifier

$letter \rightarrow (a \mid b \mid c \mid ... \mid z \mid A \mid B \mid C \mid ... \mid Z)$

$digit \rightarrow (0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9)$

$id \rightarrow letter \; (\; letter \mid digit \;)^*$

numbers

$integer \rightarrow (+ \mid - \mid \varepsilon) \; (0 \mid (1 \mid 2 \mid 3 \mid ... \mid 9) \; digit^*)$

$decimal \rightarrow integer \; \texttt{.} \; (\; digit \;)^*$

$real \rightarrow (\; integer \mid decimal \;) \; \texttt{E} \; (+ \mid -) \; digit^*$

$complex \rightarrow \texttt{'('} \; real \; \texttt{,} \; real \; \texttt{')'}$

Numbers can get much more complicated

Most programming language tokens can be described with REs

We can use REs to build scanners automatically

Algebraic properties of REs

Axiom	Description
$r \mid s = s \mid r$	$\mid$ is commutative
$r \mid (s \mid t) = (r \mid s) \mid t$	$\mid$ is associative
$(rs)t = r(st)$	concatenation is associative
$r(s \mid t) = rs \mid rt$ $(s \mid t)r = sr \mid tr$	concatenation distributes over $\mid$
$\varepsilon r = r$ $r\varepsilon = r$	ε is the identity for concatenation
$r^* = (r \mid \varepsilon)^*$	relation between $*$ and ε
$r^{**} = r^*$	$*$ is idempotent

Examples

Let $\Sigma = \{a, b\}$

1. $a|b$ denotes $\{a, b\}$

2. $(a|b)(a|b)$ denotes $\{aa, ab, ba, bb\}$
   i.e., $(a|b)(a|b) = aa|ab|ba|bb$

3. a^* denotes $\{\varepsilon, a, aa, aaa, \ldots\}$

4. $(a|b)^*$ denotes the set of all strings of a's and b's (including ε)
   i.e., $(a|b)^* = (a^*b^*)^*$

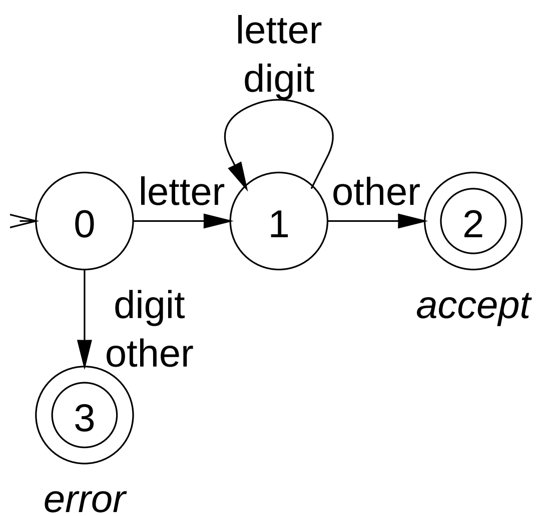
5. $a|a^*b$ denotes $\{a, b, ab, aab, aaab, aaaab, \ldots\}$

Recognizers

From a regular expression we can construct a

deterministic finite automaton (DFA)

Recognizer for *identifier*:



identifier

$$\textit{letter} \rightarrow (a \mid b \mid c \mid \ldots \mid z \mid A \mid B \mid C \mid \ldots \mid Z)$$

$$\textit{digit} \rightarrow (0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9)$$

$$\textit{id} \rightarrow \textit{letter} \ (\ \textit{letter} \mid \textit{digit} \ )^*$$

Code for the recognizer

```
char ← next_char();
state ← 0;          /* code for state 0 */
done ← false;
token_value ← ""    /* empty string */
while( not done ) {
   class ← char_class[char];
   state ← next_state[class,state];
   switch(state) {
      case 1:     /* building an id */
         token_value ← token_value + char;
         char ← next_char();
         break;
      case 2:     /* accept state */
         token_type = identifier;
         done = true;
         break;
      case 3:     /* error */
         token_type = error;
         done = true;
         break;
   }
}
return token_type;
```

Tables for the recognizer

Two tables control the recognizer

`char_class:`

	$a-z$	$A-Z$	$0-9$	other
value	letter	letter	digit	other

`next_state:`

class	0	1	2	3
letter	1	1	—	—
digit	3	1	—	—
other	3	2	—	—

To change languages, we can just change tables

Automatic construction

Scanner generators automatically construct code from regular expression-like descriptions

- construct a *dfa*
- use state minimization techniques
- emit code for the scanner

  (table driven or direct code)

A key issue in automation is an interface to the parser

`lex` is a scanner generator supplied with UNIX

- emits C code for scanner
- provides macro definitions for each token
  (used in the parser)

Grammars for regular languages

Can we place a restriction on the *form* of a grammar to ensure that it describes a regular language?

Provable fact:

For any RE r, there is a grammar g such that $L(r) = L(g)$.

The grammars that generate regular sets are called *regular grammars*

Definition:

In a regular grammar, all productions have one of two forms:

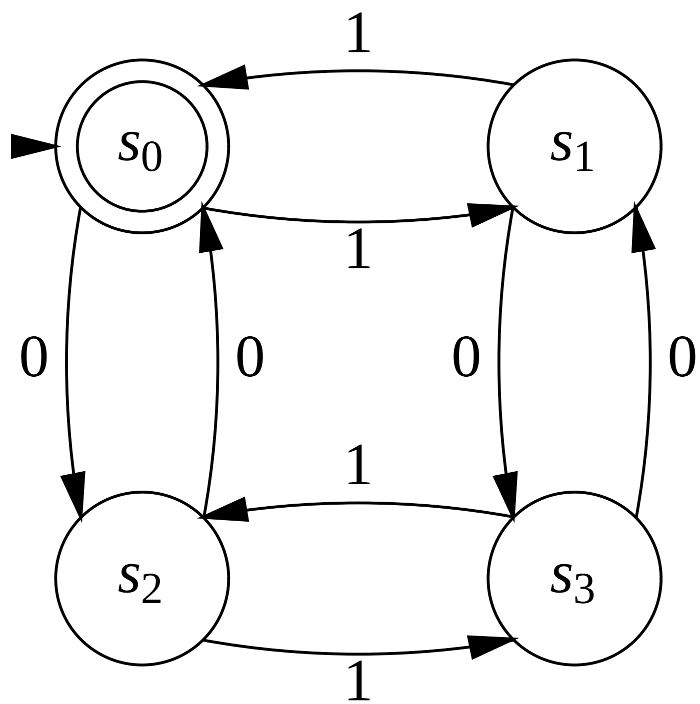
1. $A \rightarrow aA$
2. $A \rightarrow a$

where A is any non-terminal and a is any terminal symbol

These are also called *type 3* grammars (Chomsky)

More regular languages

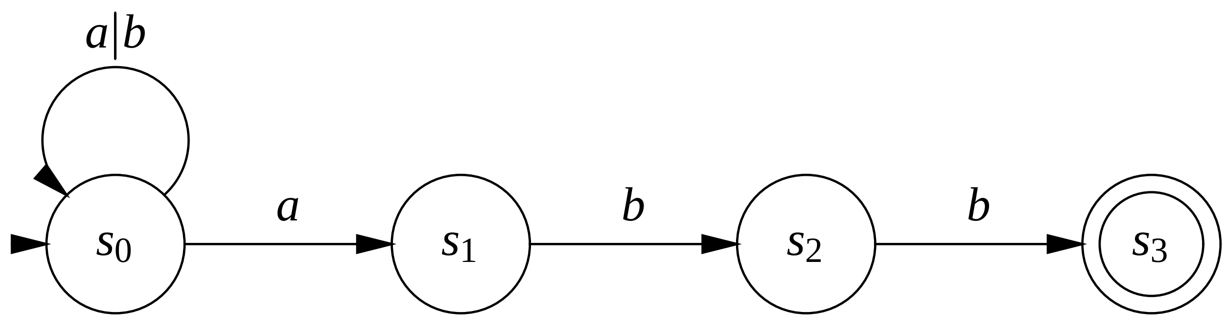
Example: the set of strings containing an even number of zeros and an even number of ones



The RE is $(00 \mid 11)^*((01 \mid 10)(00 \mid 11)^*(01 \mid 10)(00 \mid 11)^*)^*$

More regular expressions

What about the RE $(a \mid b)^* abb$?



State s_0 has multiple transitions on a!
$\Rightarrow$ *nondeterministic finite automaton*

	a	b
s_0	$\{s_0, s_1\}$	$\{s_0\}$
s_1	–	$\{s_2\}$
s_2	–	$\{s_3\}$

Finite automata

A *non-deterministic finite automaton* (NFA) consists of:

1. a set of *states* $S = \{s_0, \ldots, s_n\}$
2. a set of input symbols Σ (the alphabet)
3. a transition function *move* mapping state-symbol pairs to sets of states
4. a distinguished *start state* s_0
5. a set of distinguished *accepting* or *final* states F

A *Deterministic Finite Automaton* (DFA) is a special case of an NFA:

1. no state has a ε-transition, and
2. for each state s and input symbol a, there is at most one edge labelled a leaving s.

A DFA accepts x iff. there exists a *unique* path through the transition graph from the s_0 to an accepting state such that the labels along the edges spell x.

DFAs and NFAs are equivalent

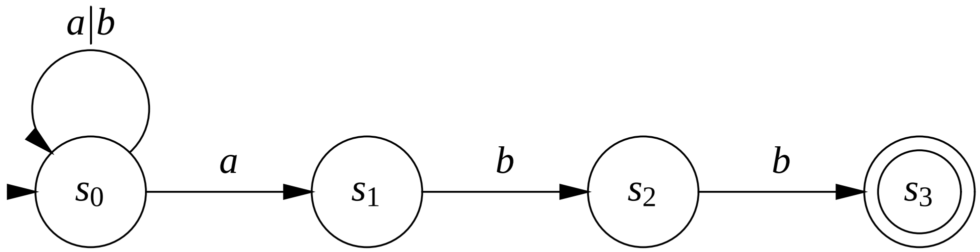
1. DFAs are clearly a subset of NFAs

2. Any NFA can be converted into a DFA, by simulating sets of simultaneous states:

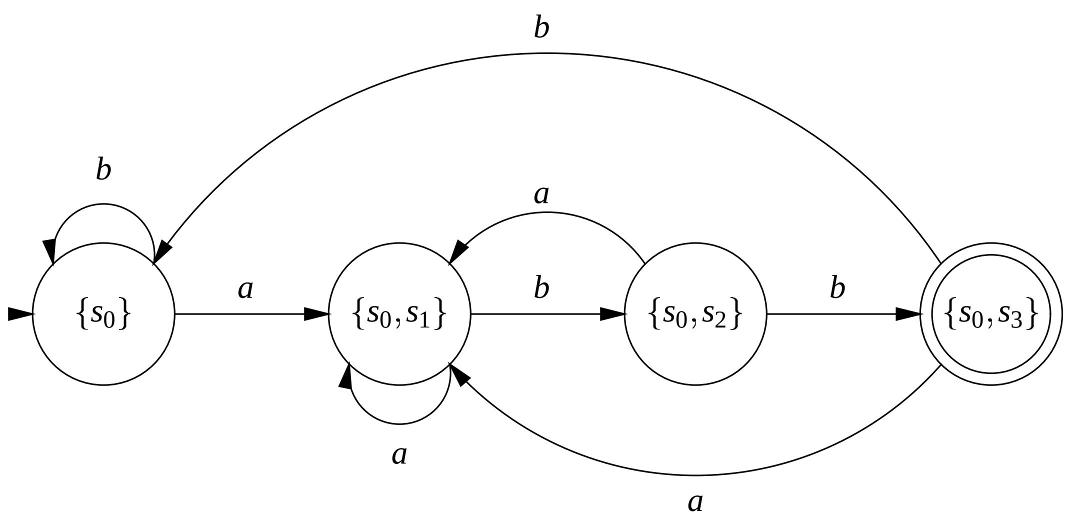
   - each DFA state corresponds to a set of NFA states

   - possible exponential blowup

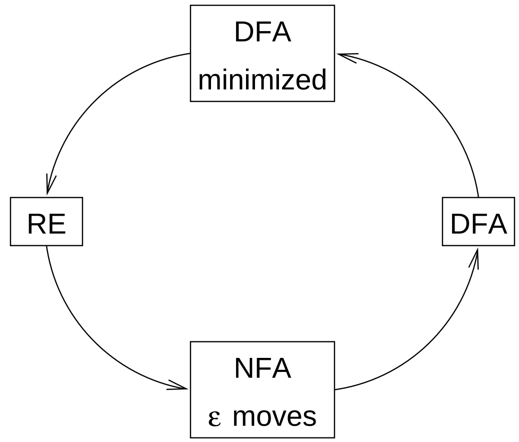
NFA to DFA using the subset construction: example 1



	a	b
$\{s_0\}$	$\{s_0, s_1\}$	$\{s_0\}$
$\{s_0, s_1\}$	$\{s_0, s_1\}$	$\{s_0, s_2\}$
$\{s_0, s_2\}$	$\{s_0, s_1\}$	$\{s_0, s_3\}$
$\{s_0, s_3\}$	$\{s_0, s_1\}$	$\{s_0\}$



Constructing a DFA from a regular expression



RE $\rightarrow$NFA w/ε moves
 build NFA for each term
 connect them with ε moves

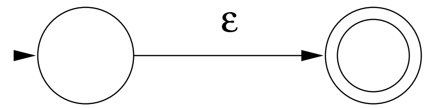
NFA w/ε moves to DFA
 construct the simulation
 the "subset" construction

DFA $\rightarrow$ minimized DFA
 merge compatible states

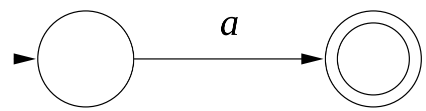
DFA $\rightarrow$ RE
 construct $R_{ij}^{k} = R_{ik}^{k-1}(R_{kk}^{k-1})^{*}R_{kj}^{k-1} \bigcup R_{ij}^{k-1}$

RE to NFA

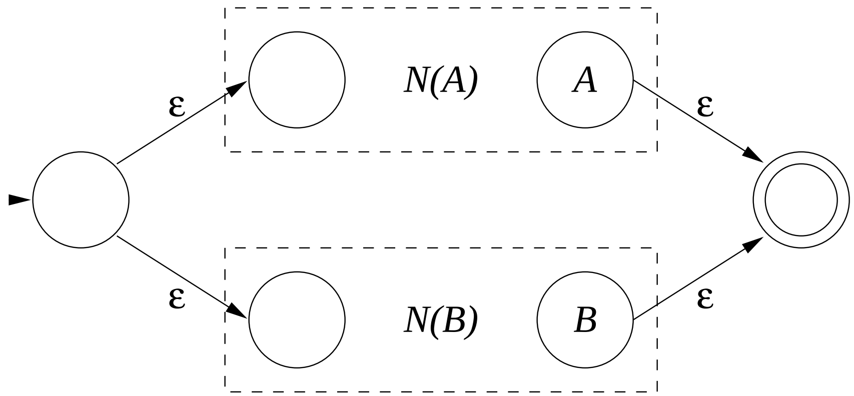
$N(\varepsilon)$



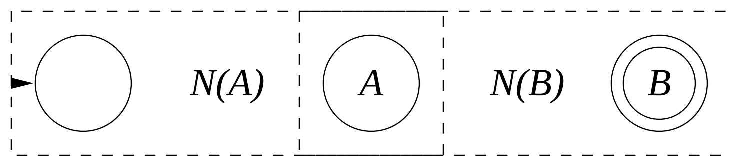
$N(a)$



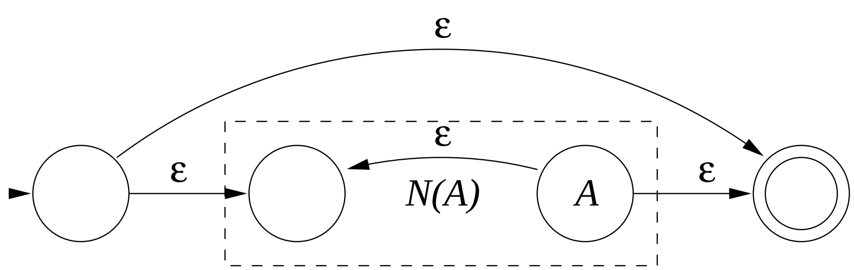
$N(A|B)$



$N(AB)$

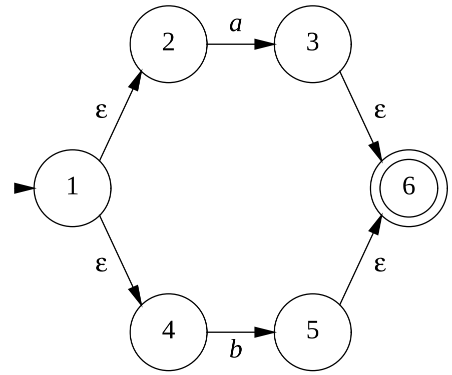


$N(A^*)$

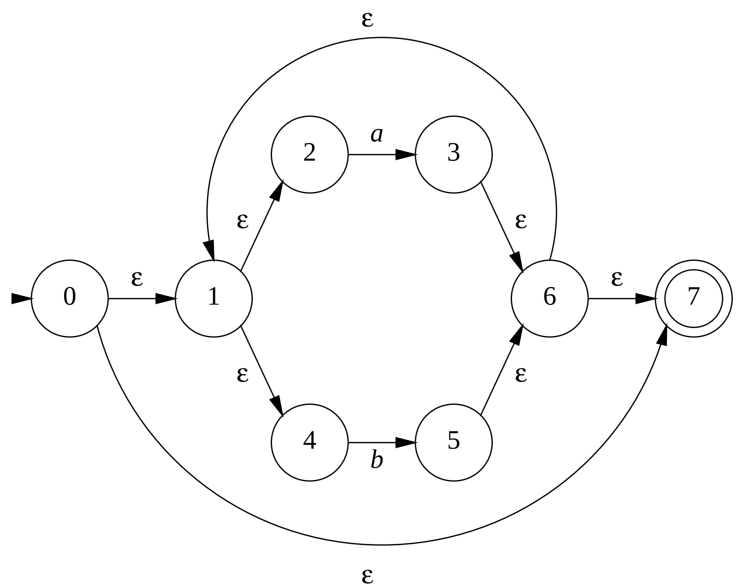


RE to NFA: example

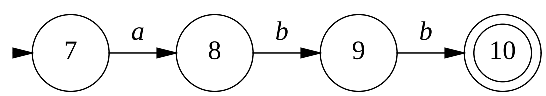
$(a \mid b)^{*}abb$



$a|b$



$(a|b)^{*}$



abb

NFA to DFA: the subset construction

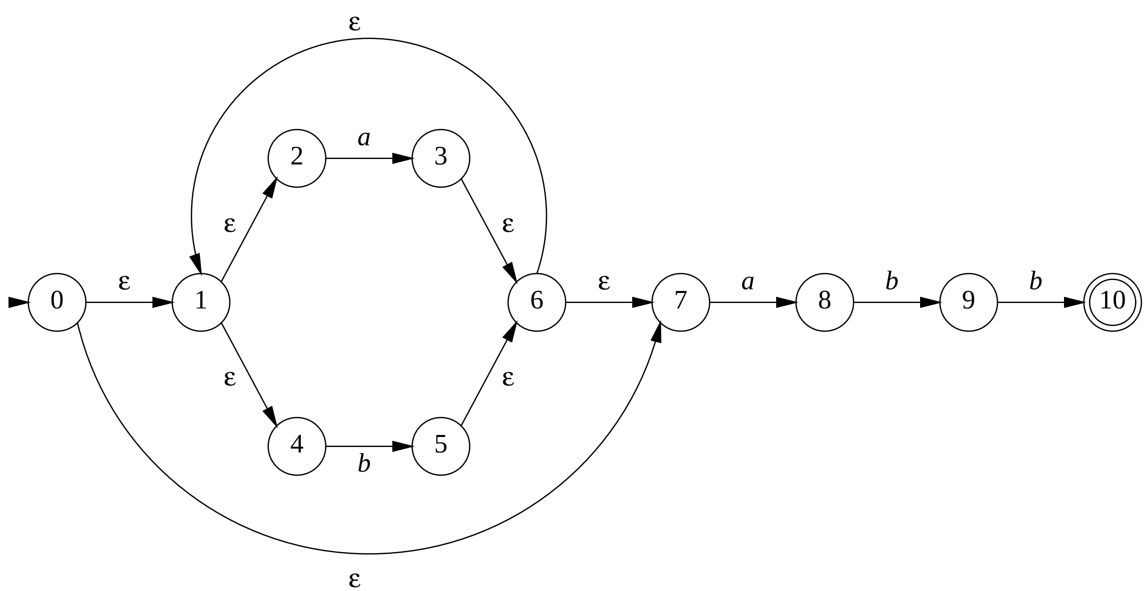
Input: NFA N
Output: A DFA D with states *Dstates* and transitions *Dtrans* such that $L(D) = L(N)$
Method: Let s be a state in N and T be a set of states, and using the following operations:

Operation	Defi nition
ε-*closure*(s)	set of NFA states reachable from NFA state s on ε-transitions alone
ε-*closure*(T)	set of NFA states reachable from some NFA state s in T on ε-transitions alone
move(T, a)	set of NFA states to which there is a transition on input symbol a from some NFA state s in T

add state $T = \varepsilon$-*closure*(s_0) unmarked to *Dstates*
while $\exists$ unmarked state T in *Dstates*
    mark T
    for each input symbol a
        $U = \varepsilon$-*closure*(*move*(T, a))
        if $U \notin$ *Dstates* **then** add U to *Dstates* unmarked
        Dtrans$[T, a] = U$
    endfor
endwhile

ε-*closure*(s_0) is the start state of D
A state of D is accepting if it contains at least one accepting state in N

NFA to DFA using subset construction: example 2



$$A = \{0,1,2,4,7\} \qquad D = \{1,2,4,5,6,7,9\}$$
$$B = \{1,2,3,4,6,7,8\} \quad E = \{1,2,4,5,6,7,10\}$$
$$C = \{1,2,4,5,6,7\}$$

	a	b
A	B	C
B	B	D
C	B	C
D	B	E
E	B	C

Limits of regular languages

Not all languages are regular

One cannot construct DFAs to recognize these languages:

- $L = \{p^k q^k\}$
- $L = \{wcw^r \mid w \in \Sigma^*\}$

Note: neither of these is a regular expression!
(DFAs cannot count!)

But, this is a little subtle. One can construct DFAs for:

- alternating 0's and 1's
  $(\varepsilon \mid 1)(01)^*(\varepsilon \mid 0)$
- sets of pairs of 0's and 1's
  $(01 \mid 10)^+$

So what is hard?

Language features that can cause problems:

reserved words

PL/I had no reserved words

```
if then then then = else; else else = then;
```

signifi cant blanks

FORTRAN and Algol68 ignore blanks

```
do 10 i = 1,25
do 10 i = 1.25
```

string constants

special characters in strings

`newline`, `tab`, `quote`, `comment delimiter`

fi nite closures

some languages limit identifier lengths

adds states to count length

FORTRAN 66 $\rightarrow$ 6 characters

These can be swept under the rug in the language design

How bad can it get?

```
1            INTEGERFUNCTIONA
2            PARAMETER(A=6,B=2)
3            IMPLICIT CHARACTER*(A-B)(A-B)
4            INTEGER FORMAT(10),IF(10),DO9E1
5     100    FORMAT(4H)=(3)
6     200    FORMAT(4 )=(3)
7            DO9E1=1
8            DO9E1=1,2
9               IF(X)=1
10              IF(X)H=1
11              IF(X)300,200
12    300       CONTINUE
13           END
      C      this is a comment
           $ FILE(1)
14           END
```

Example due to Dr. F.K. Zadeck of IBM Corporation