

## HW 4

1. Draw expression trees for the following  $\lambda$ -expressions:

- a.  $\lambda x.(x \lambda y.(y x))$
- b.  $\lambda x.\lambda y.((\lambda x.y x p)(\lambda z.z x))$

2. Make all parentheses explicit in the following  $\lambda$ -expressions:

- a.  $(\lambda p.p z) \lambda q.w \lambda w.w q z p$
- b.  $\lambda p.p q \lambda p.q p$

3. For each of the following terms, identify the free variables in each term and for each bound variable indicate (by drawing an arrow) to the  $\lambda$  to which it is bound.

- a.  $\lambda s.s z \lambda q.s q$
- b.  $(\lambda s.s z) \lambda q. w \lambda w.w q z s$

4. Apply  $\beta$ -reductions to the following  $\lambda$  expressions as much as possible:

- a.  $(\lambda z.z) (\lambda z.z z) (\lambda z.z q)$
- b.  $(\lambda s.\lambda q.s q q) (\lambda q.q) q$
- c.  $((\lambda s.s s) (\lambda q.q)) (\lambda q.q)$

5. Consider the following definitions for the booleans "true" and "false" and logical operator "and":

$\text{true} = \lambda x.\lambda y.x$   
 $\text{false} = \lambda x.\lambda y.y$   
 $\text{and} = \lambda b1.\lambda b2.(b1 b2 \text{ false})$

Using  $\beta$ -reductions, show that

- a.  $(\text{and false true})$  reduces to false
- b.  $(\text{and true true})$  reduces to true

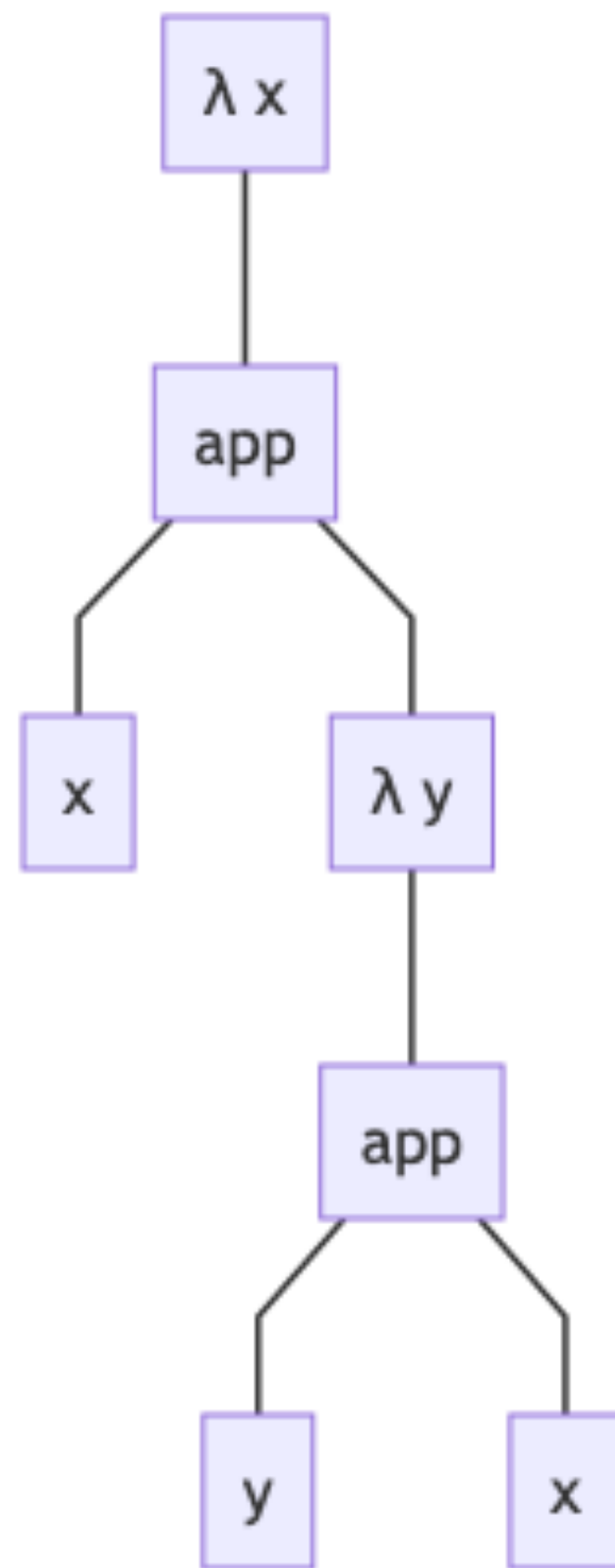
## Homework 4 SOLUTIONS

(a)

$\lambda x. (x \ \lambda y. (y \ x))$

Fully parenthesized expression:

$(\lambda x. (x \ (\lambda y. (y \ x))))$



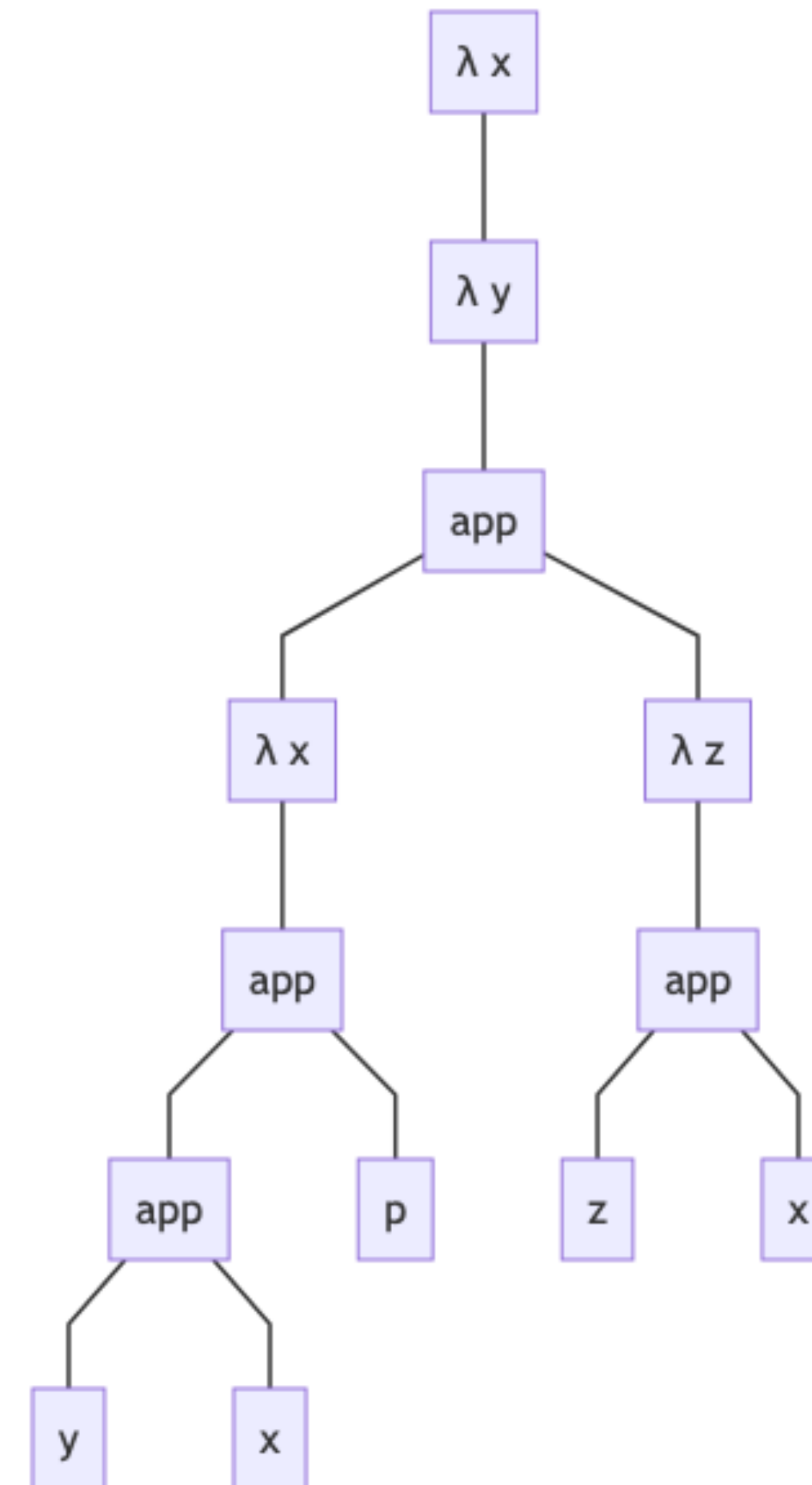
## PROBLEM 1

(b)

$\lambda x. \lambda y. ((\lambda x. y \ x \ p) (\lambda z. z \ x))$

Fully parenthesized expression:

$(\lambda x. (\lambda y. ((\lambda x. ((y \ x) \ p)) (\lambda z. (z \ x))))))$



## PROBLEM 2

(a)  $(\lambda p.p\ z)\ \lambda q.w\ \lambda w.w\ q\ z\ p$

Fully parenthesized expression:

$((\lambda p.(p\ z))\ (\lambda q.(w\ (\lambda w.(((w\ q)\ z)\ p))))))$

(b)  $\lambda p.p\ q\ \lambda p.q\ p$

Fully parenthesized expression:

$(\lambda p.((p\ q)\ (\lambda p.(q\ p))))$

## PROBLEM 3

(a)

$\lambda s.s\ z\ \lambda q.s\ q$   
 $b\ b\ f\ \ b\ b\ b$

OR in the fully parenthesized version:

$(\lambda s.((s\ z)\ (\lambda q.(s\ q))))$   
 $b\ \ \ b\ f\ \ \ \ b\ \ b\ b$

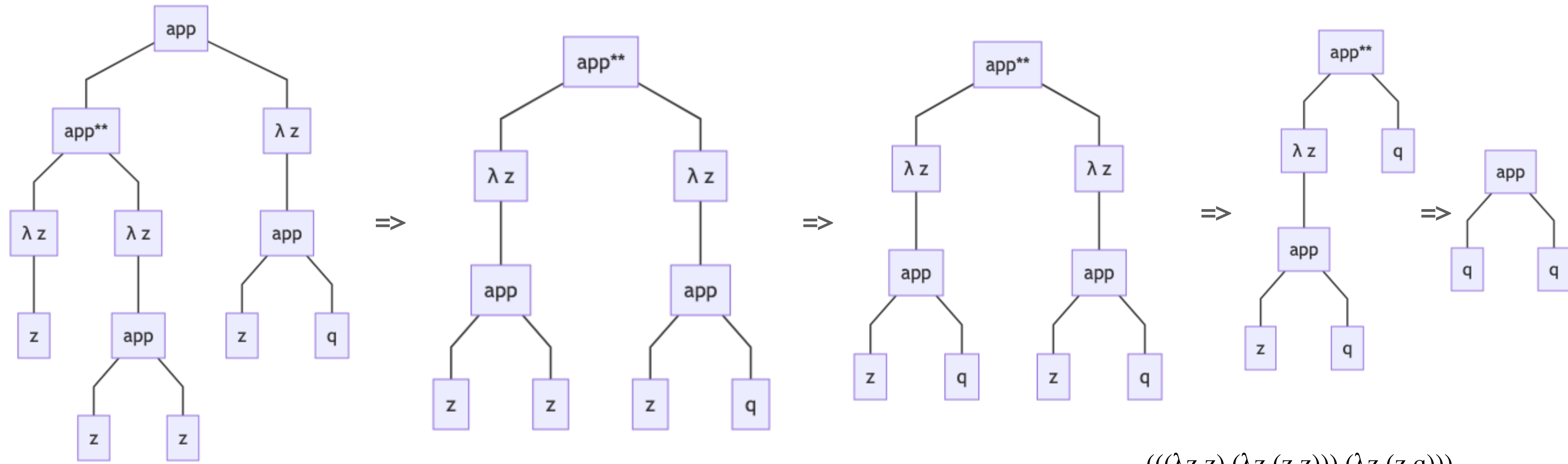
(b)

$(\lambda s.s\ z)\ \lambda q.w\ \lambda w.w\ q\ z\ s$   
 $b\ b\ f\ \ \ b\ \ f\ \ b\ b\ b\ f\ f$

OR in the fully parenthesized version:

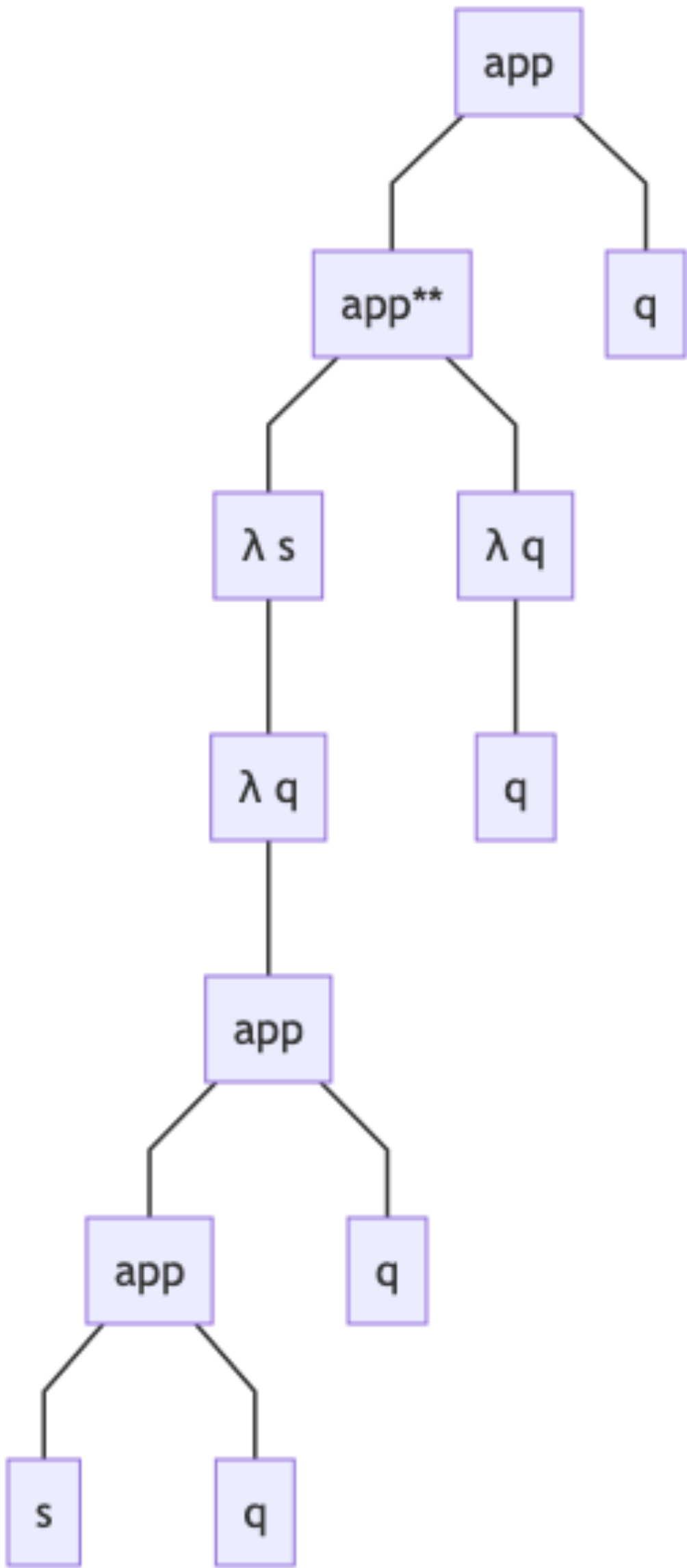
$((\lambda s.(s\ z))\ (\lambda q.\ (w\ (\lambda w.(((w\ q)\ z)\ s))))))$   
 $b\ \ b\ f\ \ \ \ b\ \ \ f\ \ \ b\ \ \ \ b\ b\ f\ \ f$

PROBLEM 4a

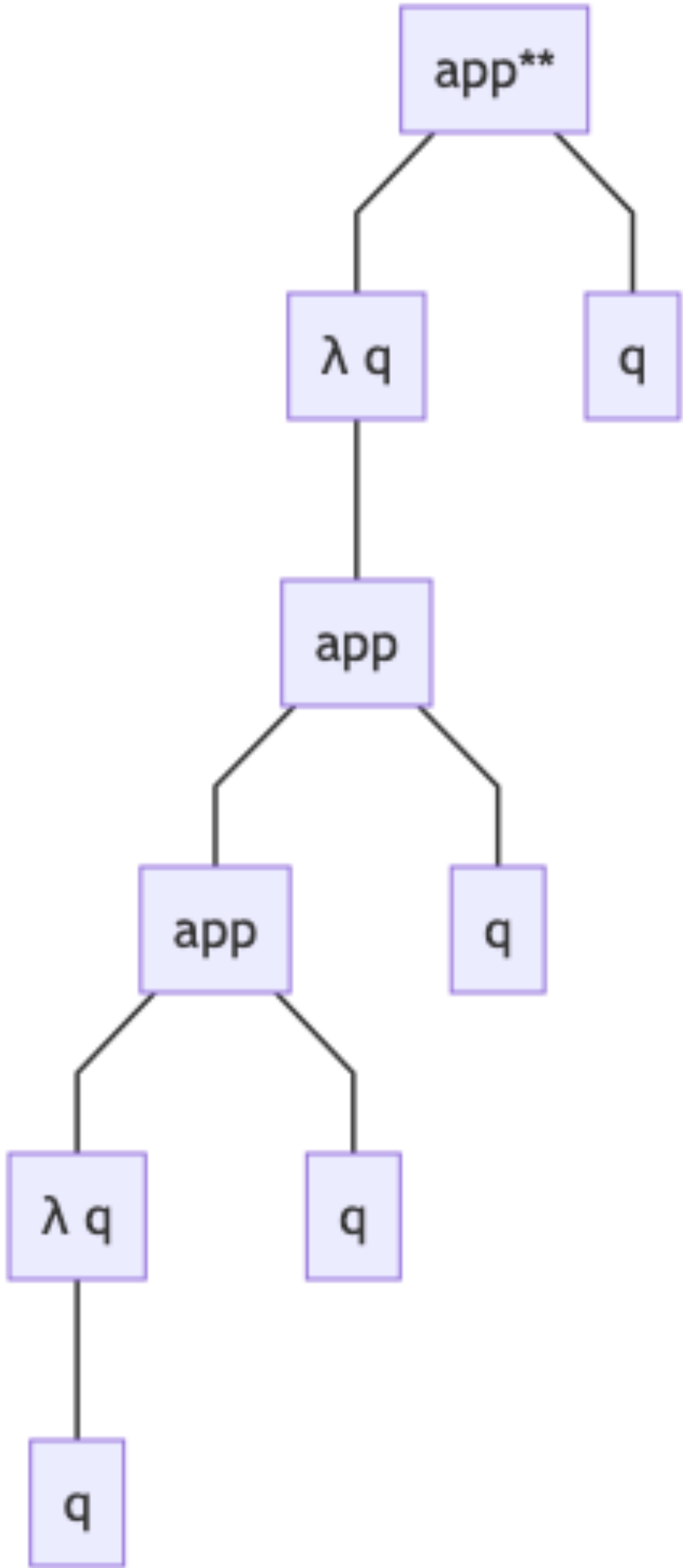


$(((\lambda z.z) (\lambda z.(z z))) (\lambda z.(z q)))$   
 $\Rightarrow ((\lambda z.(z z)) (\lambda z.(z q)))$   
 $\Rightarrow ( (\lambda z.(z q)) (\lambda z.(z q)))$   
 $\Rightarrow ( (\lambda z.(z q)) q)$   
 $\Rightarrow (q q)$

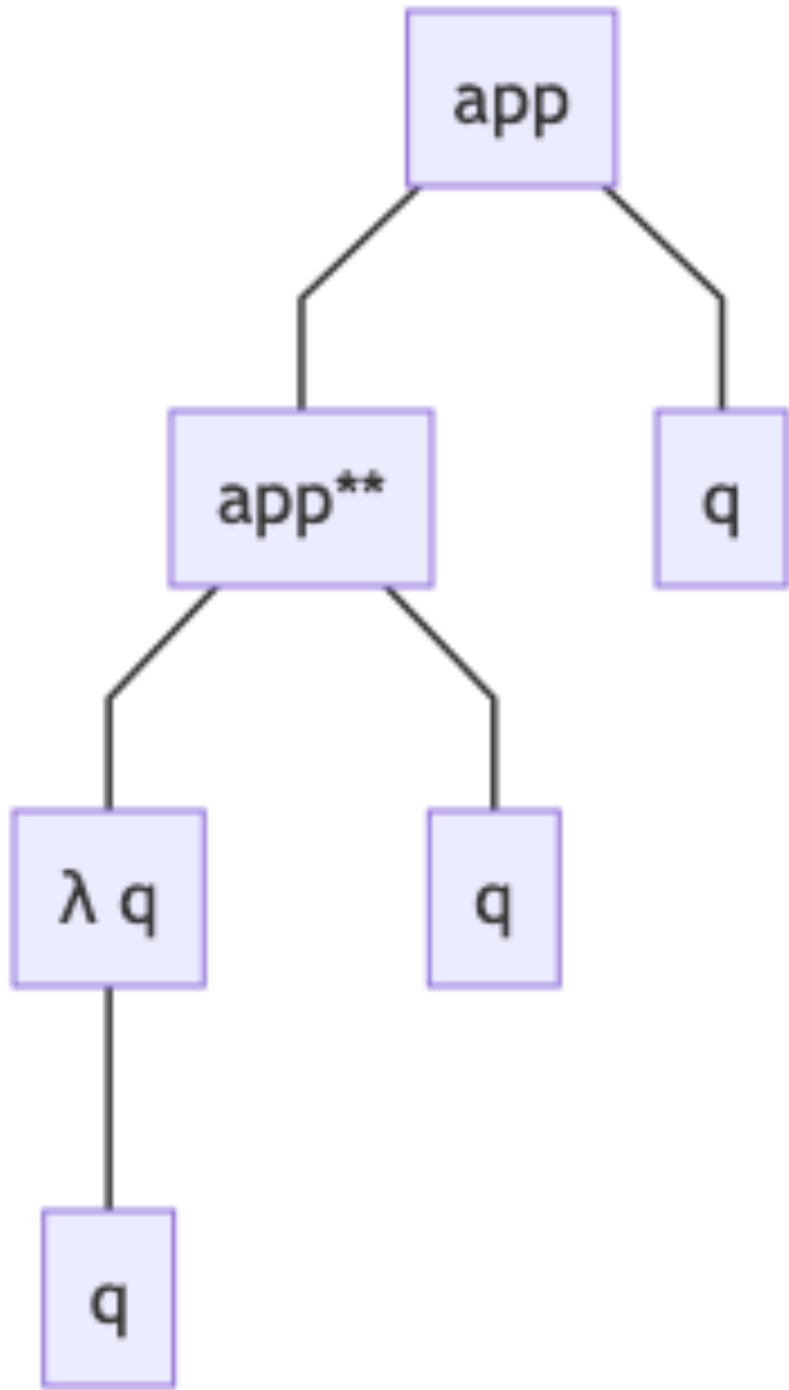
PROBLEM 4b



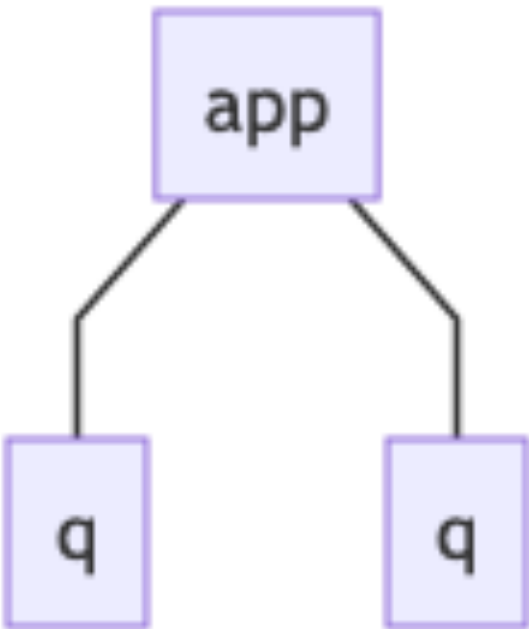
$\Rightarrow$



$\Rightarrow$

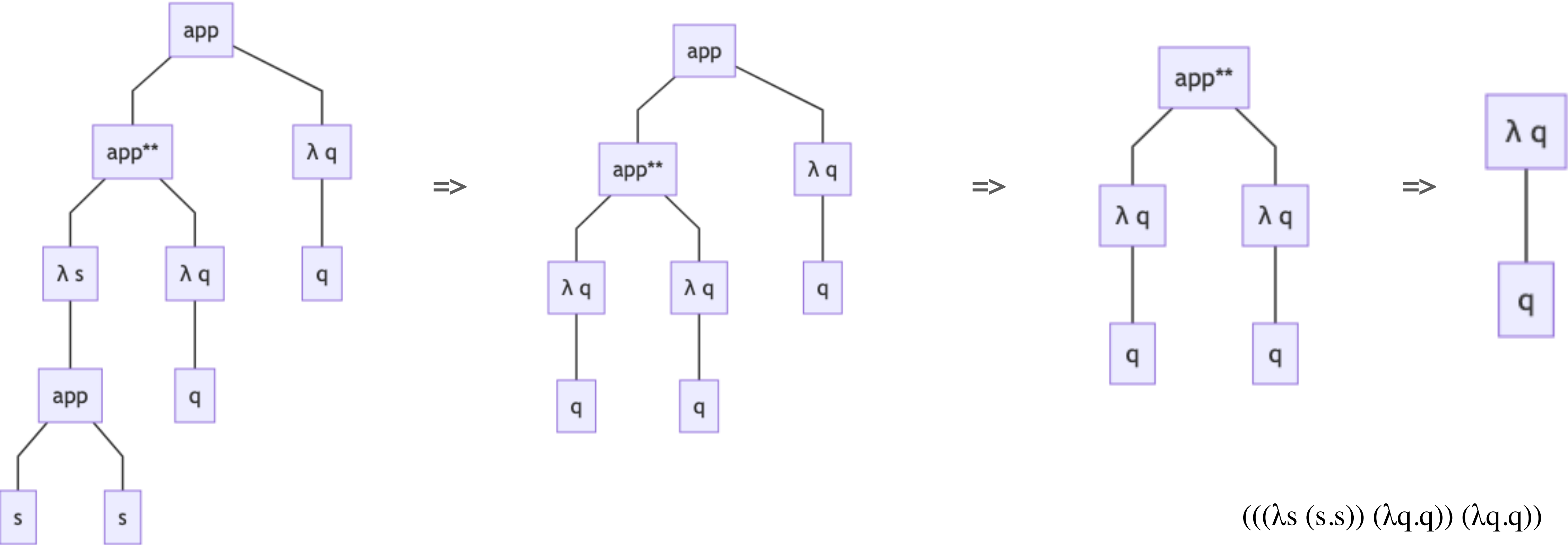


$\Rightarrow$



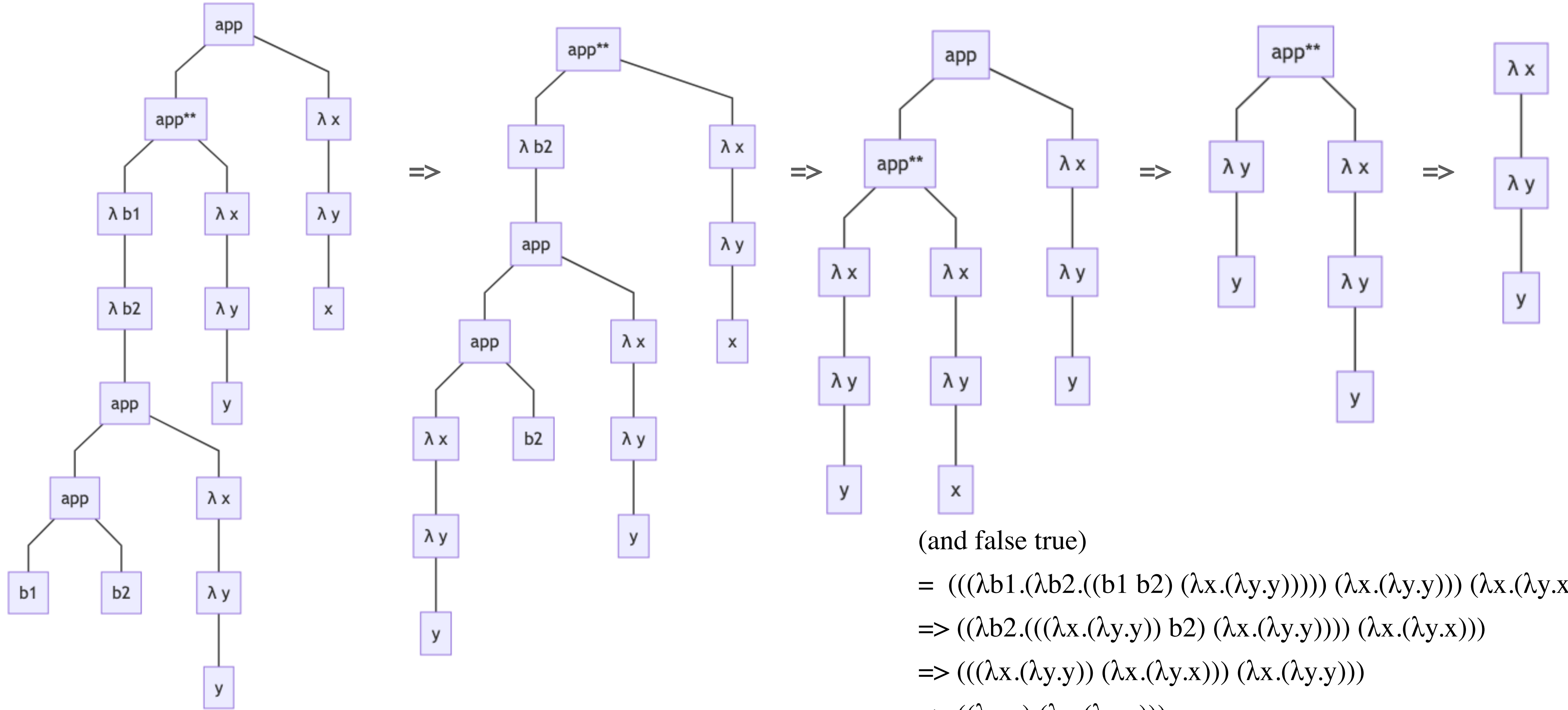
$((\lambda s. (\lambda q. ((s\ q)\ q))) (\lambda q. q))\ q$   
 $\Rightarrow ((\lambda q. (((\lambda q. q)\ q)\ q))\ q)$   
 $\Rightarrow (((\lambda q. q)\ q)\ q)$   
 $\Rightarrow (q\ q)$

PROBLEM 4c



$(((\lambda s (s.s)) (\lambda q.q)) (\lambda q.q))$   
 $\Rightarrow (((\lambda q.q) (\lambda q.q)) (\lambda q.q))$   
 $\Rightarrow ((\lambda q.q) (\lambda q.q))$   
 $\Rightarrow (\lambda q.q)$

## PROBLEM 5a



(and false true)

$= (((\lambda b1.(\lambda b2.((b1\ b2)\ (\lambda x.(\lambda y.y))))))\ (\lambda x.(\lambda y.y)))\ (\lambda x.(\lambda y.x)))$

$\Rightarrow ((\lambda b2.(((\lambda x.(\lambda y.y))\ b2)\ (\lambda x.(\lambda y.y))))\ (\lambda x.(\lambda y.x)))$

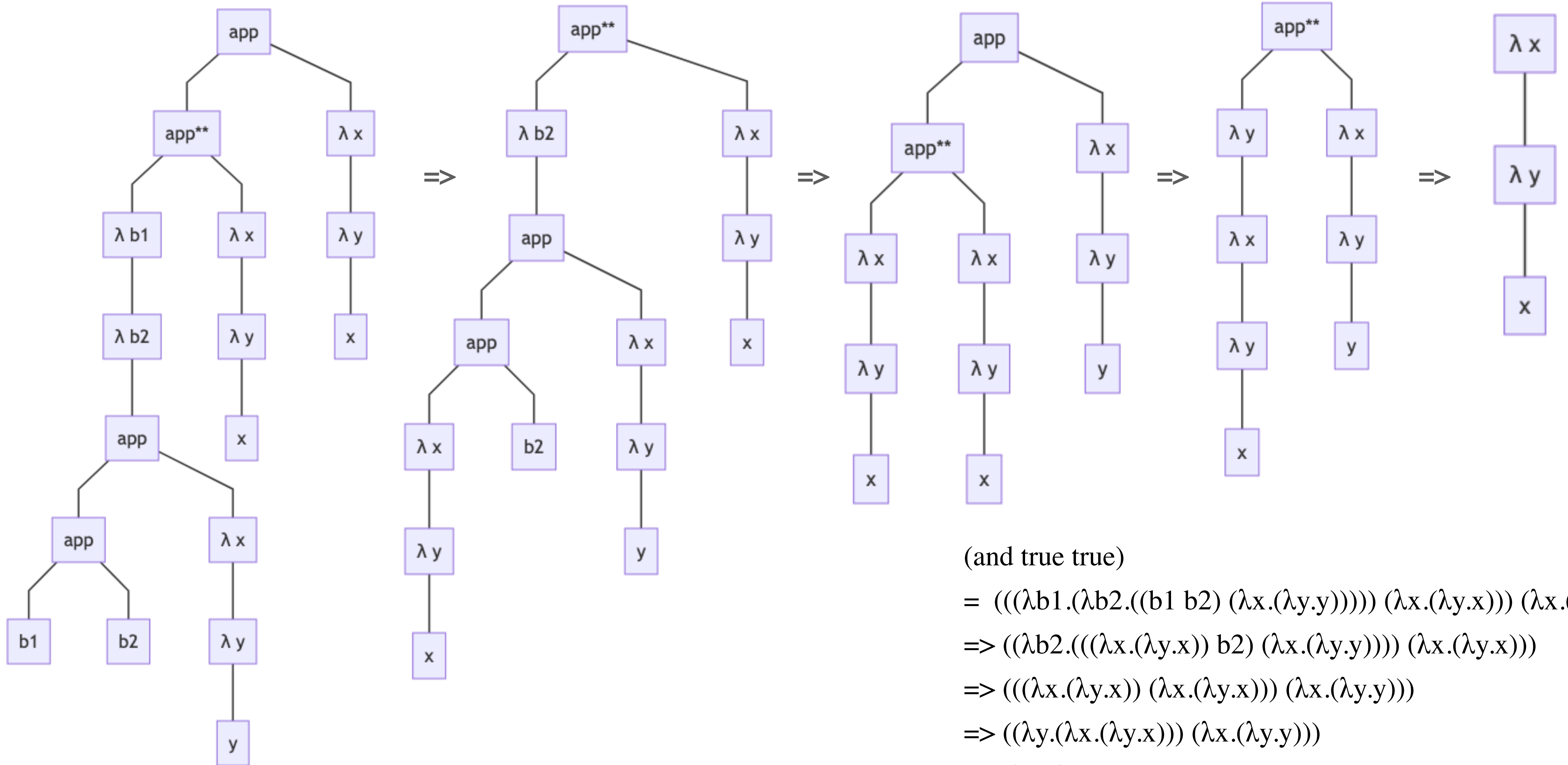
$\Rightarrow (((\lambda x.(\lambda y.y))\ (\lambda x.(\lambda y.x)))\ (\lambda x.(\lambda y.y)))$

$\Rightarrow ((\lambda y.y)\ (\lambda x.(\lambda y.y)))$

$\Rightarrow (\lambda x.(\lambda y.y))$

$= \text{false}$

## PROBLEM 5b



(and true true)

$= (((\lambda b1.(\lambda b2.((b1\ b2)\ (\lambda x.(\lambda y.y))))))\ (\lambda x.(\lambda y.x)))\ (\lambda x.(\lambda y.x)))$

$\Rightarrow ((\lambda b2.(((\lambda x.(\lambda y.x))\ b2)\ (\lambda x.(\lambda y.y))))\ (\lambda x.(\lambda y.x)))$

$\Rightarrow (((\lambda x.(\lambda y.x))\ (\lambda x.(\lambda y.x)))\ (\lambda x.(\lambda y.y)))$

$\Rightarrow ((\lambda y.(\lambda x.(\lambda y.x)))\ (\lambda x.(\lambda y.y)))$

$\Rightarrow (\lambda x.(\lambda y.x))$

$= \text{true}$