Lambda Calculus

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Lambda Calculus PLY Specification

```
expr :
    NUMBER
  | NAME
  | LPAREN expr expr RPAREN
  | LPAREN LAMBDA NAME expr RPAREN
  | LPAREN OP expr expr RPAREN
```

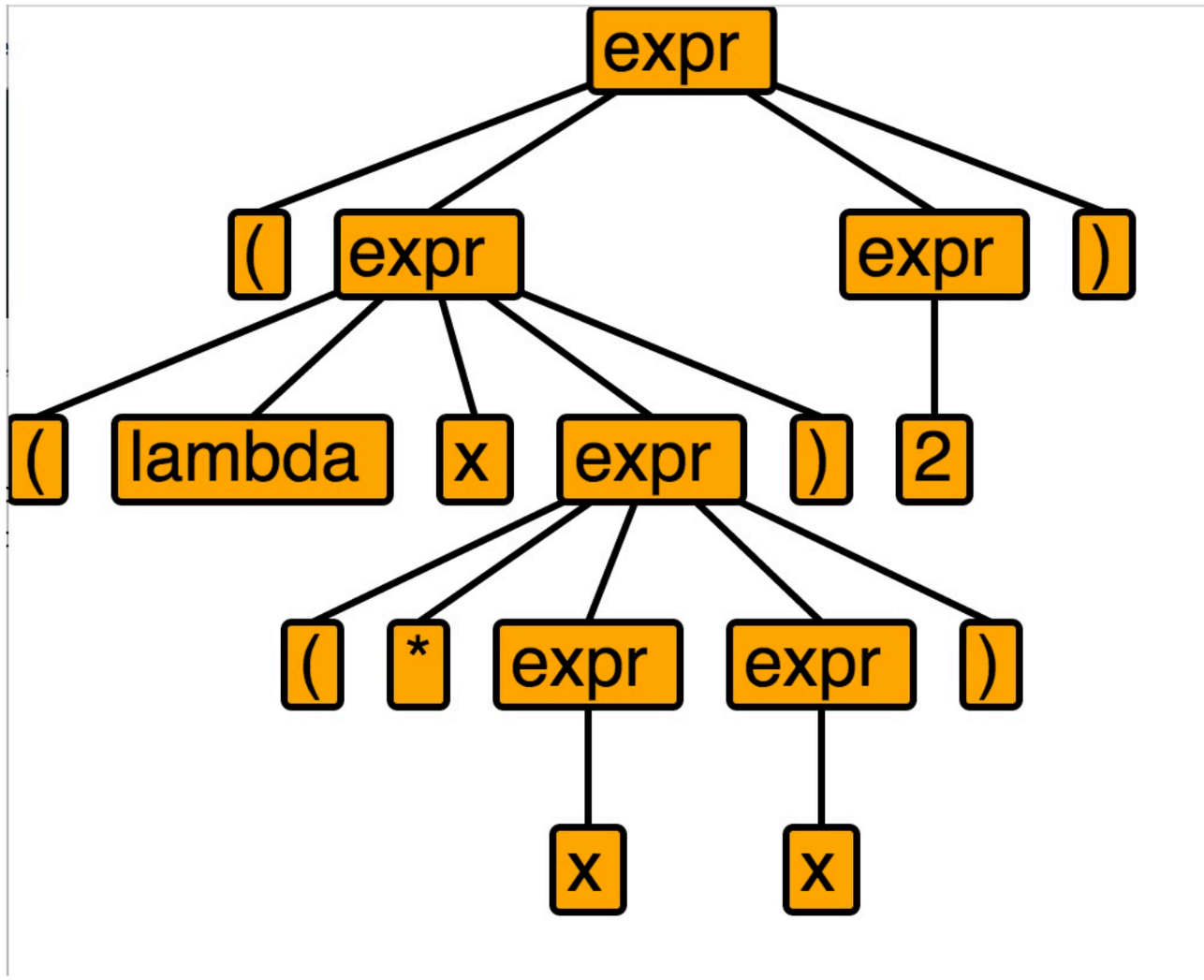
```
NUMBER = r’[0-9]+’
LPAREN = r’('
RPAREN = r’)'
OP = r’+|-|*|/‘
LAMBDA = r’[Ll][Aa][Mm][Bb][Dd][Aa]’
NAME = r‘[a-zA-z][a-zA-z0-9]*’
```

Lambda Calculus
Example 1

((lambda x (* x x)) 2)

Derivation:

expr
=> (expr expr)
=> ((lambda x expr) expr)
=> ((lambda x (* expr expr) expr)
=> ((lambda x (* x expr) expr)
=> ((lambda x (* x x) expr)
=> ((lambda x (* x x) 2)



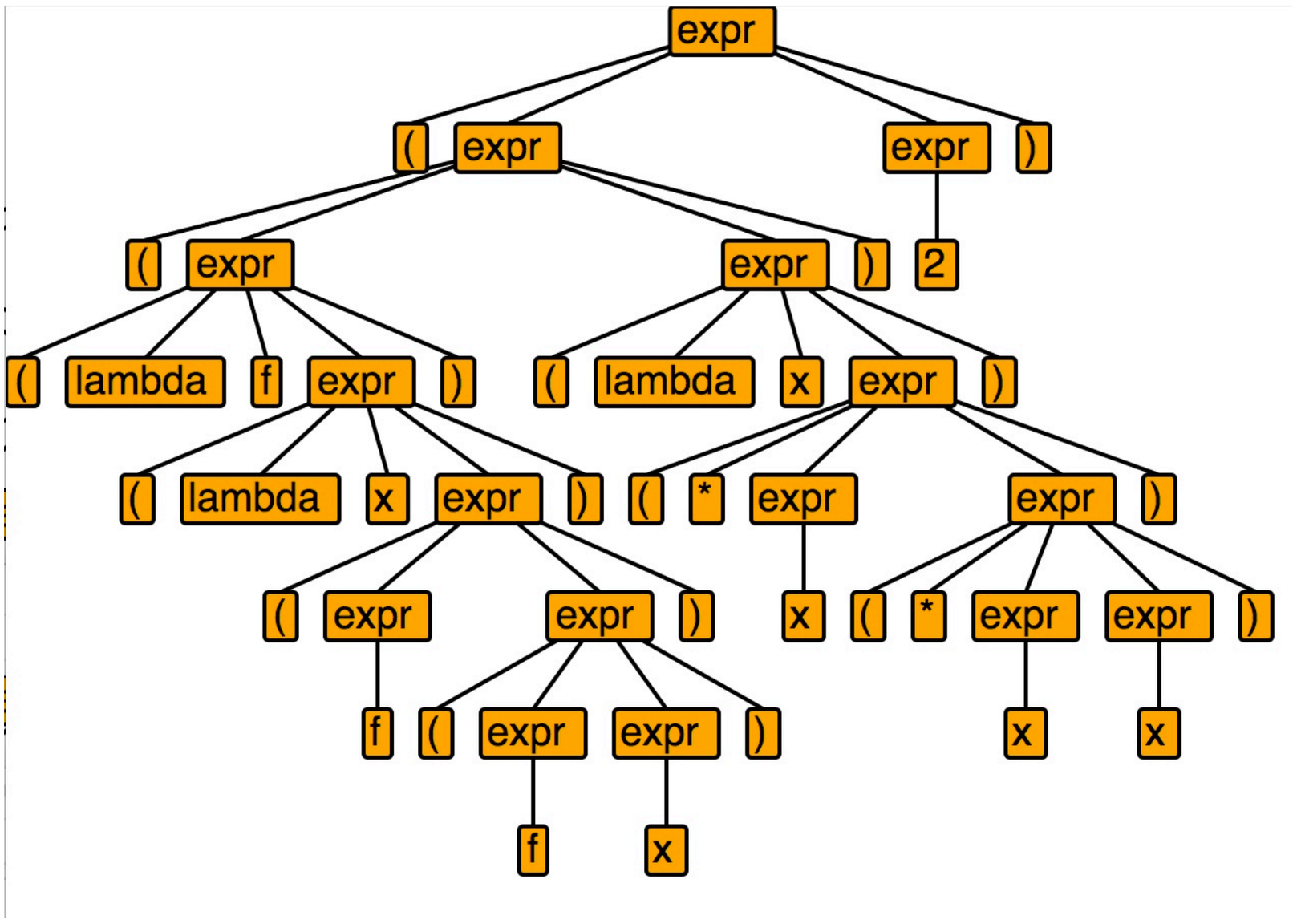
Evaluation:

((lambda x (* x x)) 2)
=beta
(* 2 2)
=math
4

Lambda Calculus
Example 2 (HOF)

```
(lambda f (lambda x (f (f x))))
```

(((lambda f (lambda x (f (f x)))) (lambda x (* x (* x x)))) 2)



Lambda Calculus
Example 2 - Evaluation

Consider the 2-parameter function:

(lambda f (lambda x (f (f x))))

application of above function:

(((lambda f (lambda x (f (f x)))) (lambda y (* y (* y y)))) 2)

=beta

((lambda x ((lambda y (* y (* y y))) ((lambda y (* y (* y y))) x))) 2)

=beta

((lambda y (* y (* y y))) ((lambda y (* y (* y y))) 2)))

=beta

((lambda y (* y (* y y))) (* 2 (* 2 2)))=math ((lambda y (* y (* y y))) 8)

=beta

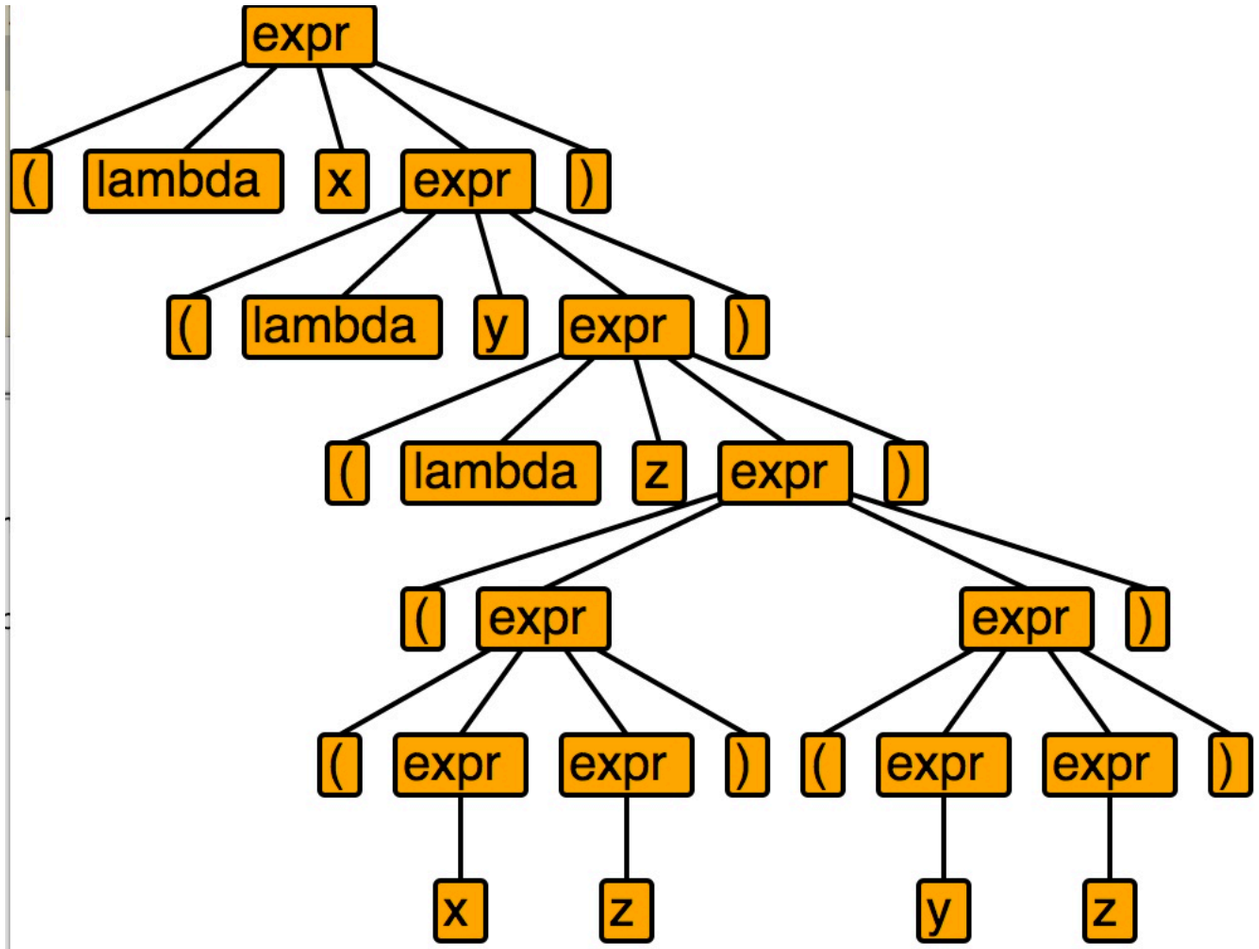
(* 8 (* 8 8)) =math 512

Lambda Calculus
Example 3

(lambda x (lambda y (lambda z (* (x z)(y z)))))

Derivation:

expr
=> (lambda x expr)
=> (lambda x (lambda y expr))
=> (lambda x (lambda y (lambda z expr)))
=> (lambda x (lambda y (lambda z (expr expr))))
=> (lambda x (lambda y (lambda z ((expr expr) expr))))
=> (lambda x (lambda y (lambda z ((expr expr) (expr expr)))))
=> (lambda x (lambda y (lambda z ((x expr) (expr expr)))))
=> (lambda x (lambda y (lambda z ((x z) (expr expr)))))
=> (lambda x (lambda y (lambda z ((x z) (y expr)))))
=> (lambda x (lambda y (lambda z ((x z) (y z)))))

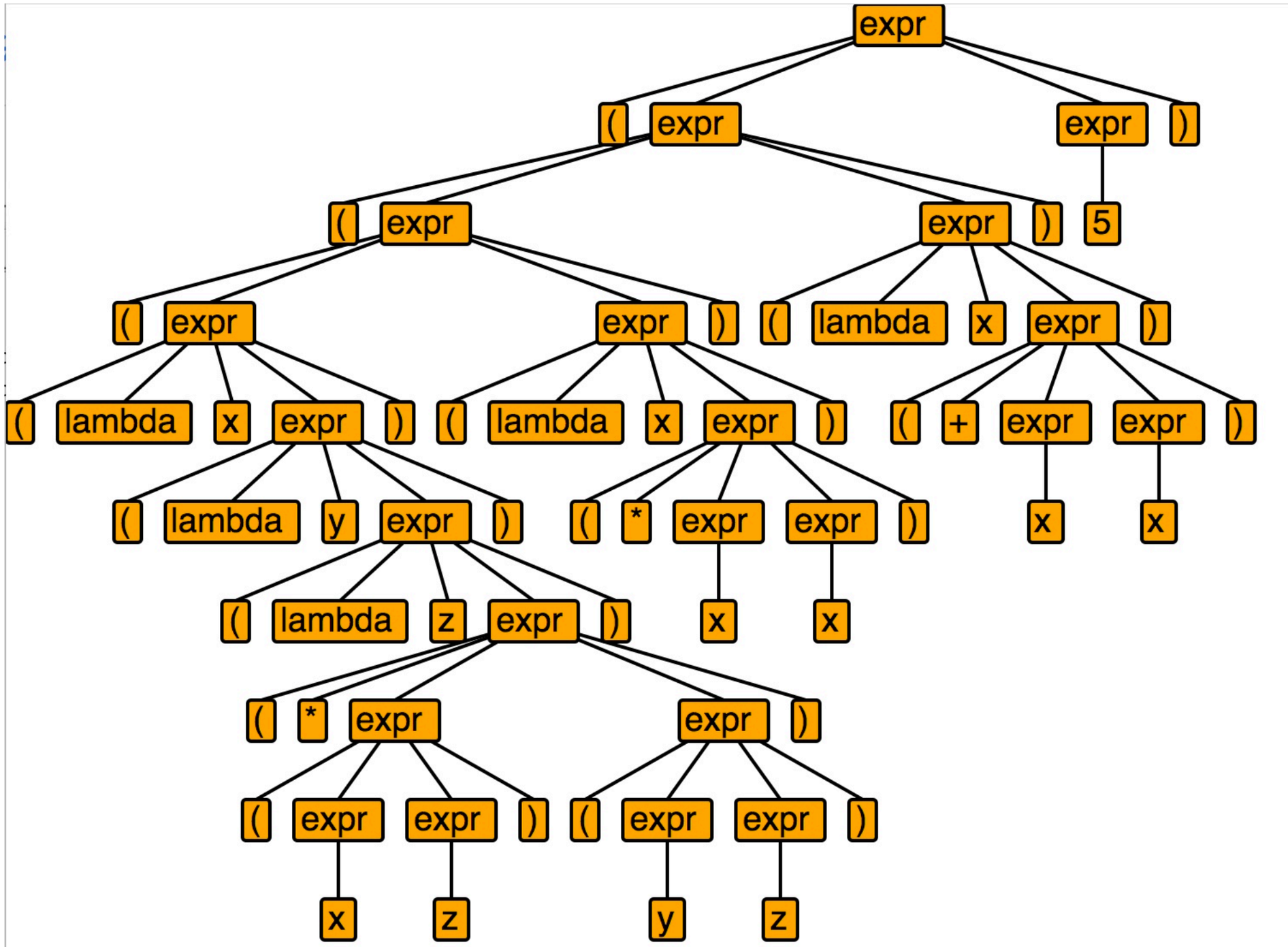


Application:

```
((( (lambda x (lambda y (lambda z (* (x z)(y z)))))
  (lambda x (* x x)))
  (lambda x (+ x x)))
  5)
```

Lambda Calculus
Example 3 - Parse Tree

```
((( (lambda x (lambda y (lambda z (* (x z) (y z)))))
(lambda x (* x x)))(lambda x (+ x x))) 5)
```



Lambda Calculus
Example 3 - Evaluation

((((lambda x (lambda y (lambda z (* (x z)(y z))))) (lambda x (* x x))) (lambda x (+ x x))) 5)
=beta