1. Let $S = \{2, 5, 17, 27\}$. Which of the following are true?
   a. $5 \in S$ b. $2 + 5 \in S$ c. $\varnothing \in S$ d. $S \in S$
2. Let $B = \{x \mid x \in \mathbb{Q} \text{ and } -1 < x < 2\}$. Which of the following are true?
   a. $0 \in B$ b. $-1 \in B$ c. $-0.84 \in B$ d. $\sqrt{2} \in B$
3. How many different sets are described in the following? What are they?

   $\{2, 3, 4\}$
   $\{x \mid x$ is the first letter of cat, bat, or apple$\}$
   $\{x \mid x \in \mathbb{N} \text{ and } 2 \leq x \leq 4\}$
   $\{a, b, e\}$
   $\varnothing$
   $\{x \mid x$ is the first letter of cat, bat, and apple$\}$
   $\{2, a, 3, b, 4, c\}$
   $\{3, 4, 2\}$

4. Describe each of the following sets by listing its elements:
   a. $\{x \mid x \in \mathbb{N} \text{ and } x^2 < 25\}$
   b. $\{x \mid x \in \mathbb{N} \text{ and } x \text{ is even and } 2 < x < 11\}$
   c. $\{x \mid x$ is one of the first three U.S. presidents$\}$
   d. $\{x \mid x \in \mathbb{R} \text{ and } x^2 = -1\}$
   e. $\{x \mid x$ is one of the New England states$\}$
   f. $\{x \mid x \in \mathbb{Z} \text{ and } |x| < 4\}$ ($|x|$ denotes the absolute value function)
5. Describe each of the following sets by listing its elements:
   a. $\{x \mid x \in \mathbb{N} \text{ and } x^2 - 5x + 6 = 0\}$
   b. $\{x \mid x \in \mathbb{R} \text{ and } x^2 = 7\}$
   c. $\{x \mid x \in \mathbb{N} \text{ and } x^2 - 2x - 8 = 0\}$
6. Describe each of the following sets by giving a characterizing property:
   a. $\{1, 2, 3, 4, 5\}$
   b. $\{1, 3, 5, 7, 9, 11, ...\}$
   c. {Melchior, Gaspar, Balthazar}
   d. $\{0, 1, 10, 11, 100, 101, 110, 111, 1000, ...\}$
7. Describe each of the following sets:
   a. $\{x \mid x \in \mathbb{N} \text{ and } (\exists q)(q \in \{2, 3\} \text{ and } x = 2q)\}$
   b. $\{x \mid x \in \mathbb{N} \text{ and } (\exists y)(\exists z)(y \in \{0, 1\} \text{ and } z \in \{3, 4\} \text{ and } y < x < z)\}$
   c. $\{x \mid x \in \mathbb{N} \text{ and } (\forall y)(y \text{ even} \to x \neq y)\}$
8. Given the description of a set A as $A = \{2, 4, 8, ...\}$, do you think $16 \in A$?
9. What is the cardinality of each of the following sets?
   a. $S = \{a, \{a, \{a\}\}\}$
   b. $S = \{\{a\}, \{\{a\}\}\}$
   c. $S = \{\varnothing\}$
   d. $S = \{a, \{\varnothing\}, \varnothing\}$
   e. $S = \{\varnothing, \{\varnothing, \{\varnothing\}\}, \{\varnothing, \{\varnothing, \{\varnothing\}\}\}\}$
10. Let

$$A = \{2, 5, 7\}$$
$$B = \{1, 2, 4, 7, 8\}$$
$$C = \{7, 8\}$$

Which of the following statements are true?
   a. $5 \subseteq A$
   b. $C \subseteq B$
   c. $\varnothing \in A$
   d. $7 \in B$
   e. $\{2, 5\} \subseteq A$
   f. $\varnothing \subseteq C$

11. Let

$$A = \{x \mid x \in \mathbb{N} \text{ and } 1 < x < 50\}$$
$$B = \{x \mid x \in \mathbb{R} \text{ and } 1 < x < 50\}$$
$$C = \{x \mid x \in \mathbb{Z} \text{ and } |x| \geq 25\}$$

Which of the following statements are true?

a. $A \subseteq B$
b. $17 \in A$
c. $A \subseteq C$
d. $-40 \in C$
e. $\sqrt{3} \in B$
f. $\{0, 1, 2\} \subseteq A$
g. $\varnothing \in B$
h. $\{x \mid x \in \mathbb{Z} \text{ and } x^2 > 625\} \subseteq C$

12. Let

$$R = \{1, 3, \pi, 4.1, 9, 10\} \qquad T = \{1, 3, \pi\}$$
$$S = \{\{1\}, 3, 9, 10\} \qquad U = \{\{1, 3, \pi\}, 1\}$$

Which of the following are true? For those that are not, why not?

★ a. $S \subseteq R$
★ b. $1 \in R$
★ c. $1 \in S$
★ d. $1 \subseteq U$
★ e. $\{1\} \subseteq T$
★ f. $\{1\} \subseteq S$
g. $T \subset R$
h. $\{1\} \in S$
i. $\varnothing \subseteq S$
j. $T \subseteq U$
k. $T \in U$
l. $T \notin R$
m. $T \subseteq R$
n. $S \subseteq \{1, 3, 9, 10\}$

13. Let

$$A = \{a, \{a\}, \{\{a\}\}\} \qquad B = \{a\} \qquad C = \{\varnothing, \{a, \{a\}\}\}$$

Which of the following are true? For those that are not, where do they fail?

a. $B \subseteq A$
b. $B \in A$
c. $C \subseteq A$
d. $\varnothing \subseteq C$
e. $\varnothing \in C$
f. $\{a, \{a\}\} \in A$
g. $\{a, \{a\}\} \subseteq A$
h. $B \subseteq C$
i. $\{\{a\}\} \subseteq A$

14. Let

$$A = \{x \mid x \in \mathbb{R} \text{ and } x^2 - 4x + 3 < 0\}$$

and

$$B = \{x \mid x \in \mathbb{R} \text{ and } 0 < x < 6\}$$

Prove that $A \subset B$.

★ 15. Let

$$A = \{(x, y) \mid (x, y) \text{ lies within 3 units of the point } (1, 4)\}$$

and

$$B = \{(x, y) \mid (x - 1)^2 + (y - 4)^2 \leq 25\}$$

Prove that $A \subset B$.

16. Program QUAD finds and prints solutions to quadratic equations of the form $ax^2 + bx + c = 0$. Program EVEN lists all the even integers from $-2n$ to $2n$. Let Q denote the set of values output by QUAD and E denote the set of values output by EVEN.

a. Show that for $a = 1$, $b = -2$, $c = -24$, and $n = 50$, $Q \subseteq E$.
b. Show that for the same values of a, b, and c, but a value for n of 2, $Q \not\subseteq E$.

17. Let $A = \{x \mid \cos(x/2) = 0\}$ and $B = \{x \mid \sin x = 0\}$. Prove that $A \subseteq B$.
18. Which of the following are true for all sets A, B, and C?
   ★ a. If $A \subseteq B$ and $B \subseteq A$, then $A = B$.
   ★ b. $\{\emptyset\} = \emptyset$
   ★ c. $\{\emptyset\} = \{0\}$
   ★ d. $\emptyset \in \{\emptyset\}$
   ★ e. $\emptyset \subseteq A$
   ★ f. $\emptyset \in A$
   g. $\{\emptyset\} = \{\{\emptyset\}\}$
   h. If $A \subset B$ and $B \subseteq C$, then $A \subset C$.
   i. If $A \neq B$ and $B \neq C$, then $A \neq C$.
   j. If $A \in B$ and $B \nsubseteq C$, then $A \notin C$.
19. Prove that if $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.
20. Prove that if $A' \subseteq B'$, then $B \subseteq A$.
21. a. Prove that for any integer $n \geq 2$, a set with n elements has $n(n - 1)/2$ subsets that contain exactly two elements.
   b. Prove that for any integer $n \geq 3$, a set with n elements has $n(n - 1)(n - 2)/6$ subsets that contain exactly three elements. (*Hint:* Use part (a).)
22. Find $\wp(S)$ for $S = \{a\}$.
★ 23. Find $\wp(S)$ for $S = \{1, 2, 3, 4\}$. How many elements do you expect this set to have?
24. Find $\wp(S)$ for $S = \{\emptyset\}$.
25. Find $\wp(S)$ for $S = \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$.
26. Find $\wp(\wp(S))$ for $S = \{a, b\}$.
★ 27. What can be said about A if $\wp(A) = \{\emptyset, \{x\}, \{y\}, \{x, y\}\}$?
28. What can be said about A if $\wp(A) = \{\emptyset, \{a\}, \{\{a\}\}\}$?
29. Prove that if $A \subseteq B$, then $\wp(A) \subseteq \wp(B)$.
★ 30. Prove that if $\wp(A) = \wp(B)$, then $A = B$.
31. Solve for x and y.
   a. $(y, x + 2) = (5, 3)$ b. $(2x, y) = (16, 7)$ c. $(2x - y, x + y) = (-2, 5)$
32. a. Recall that ordered pairs must have the property that $(x, y) = (u, v)$ if and only if $x = u$ and $y = v$. Prove that $\{\{x\}, \{x, y\}\} = \{\{u\}, \{u, v\}\}$ if and only if $x = u$ and $y = v$. Therefore, although we know that $(x, y) \neq \{x, y\}$, we can define the ordered pair (x, y) as the set $\{\{x\}, \{x, y\}\}$.
   b. Show by an example that we cannot define the ordered triple (x, y, z) as the set $\{\{x\}, \{x, y\}, \{x, y, z\}\}$
33. Which of the following are binary or unary operations on the given sets? For those that are not, where do they fail?
   ★ a. $x \circ y = x + 1$; $S = \mathbb{N}$
   ★ b. $x \circ y = x + y - 1$; $S = \mathbb{N}$
   ★ c. $x \circ y = \begin{cases} x - 1 & \text{if } x \text{ is odd} \\ x & \text{if } x \text{ is even} \end{cases}$ $S = \mathbb{Z}$
   d. $x^{\#} = \ln x$; $S = \mathbb{R}$
   e. $x^{\#} = x^2$; $S = \mathbb{Z}$
   f. $S = \{1, 2, 3\}$

∘	1	2	3
1	1	2	3
2	2	3	4
3	3	4	5

   g. $x \circ y$ = that fraction, x or y, with the smaller denominator; S = set of all fractions
   h. $x \circ y$ = that person, x or y, whose name appears first in an alphabetical sort; S = set of 10 people with different names

34. Which of the following are binary or unary operations on the given sets? For those that are not, where do they fail?

a. $x \circ y = \begin{cases} 1/x & \text{if } x \text{ is positive} \\ 1/(-x) & \text{if } x \text{ is negative} \end{cases} \quad S = \mathbb{R}$

b. $x \circ y = xy$ (concatenation); S = set of all finite-length strings of symbols from the set $\{p, q, r\}$

c. $x^{\#} = \lfloor x \rfloor$ where $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x; $S = \mathbb{R}$

d. $x \circ y = \min(x, y)$; $S = \mathbb{N}$

e. $x \circ y$ = greatest common multiple of x and y; $S = \mathbb{N}$

f. $x \circ y = x + y$; S = the set of Fibonacci numbers

g. $x^{\#}$ = the string that is the reverse of x; S = set of all finite-length strings of symbols from the set $\{p, q, r\}$

h. $x \circ y = x + y$; $S = \mathbb{R} - \mathbb{Q}$

35. How many different binary operations can be defined on a set with n elements? (*Hint:* Think about filling in a table.)

36. We have written binary operations in *infix* notation, where the operation symbol appears between the two operands, as in $A + B$. Evaluation of a complicated arithmetic expression is more efficient when the operations are written in *postfix* notation, where the operation symbol appears after the two operands, as in $AB +$. Many compilers change expressions in a computer program from infix to postfix form. One way to produce an equivalent postfix expression from an infix expression is to write the infix expression with a full set of parentheses, move each operator to replace its corresponding right parenthesis, and then eliminate all left parentheses. (Parentheses are not required in postfix notation.) Thus,

$$A * B + C$$

becomes, when fully parenthesized,

$$((A * B) + C)$$

and the postfix notation is $AB * C+$. Rewrite each of the following in postfix notation:

a. $(A + B) * (C - D)$

b. $A ** B - C * D$ (**denotes exponentiation)

c. $A * C + B/(C + D * B)$

★ 37. Evaluate the following postfix expressions (see Exercise 36):

a. $2\ 4 * 5 +$ b. $5\ 1 + 2 / 1 -$ c. $3\ 4 + 5\ 1 - *$

38. Let

$$A = \{p, q, r, s\}$$
$$B = \{r, t, v\}$$
$$C = \{p, s, t, u\}$$

be subsets of $S = \{p, q, r, s, t, u, v, w\}$. Find

a. $B \cap C$
b. $A \cup C$
c. C'
d. $A \cap B \cap C$
e. $B - C$
f. $(A \cup B)'$
g. $A \times B$
h. $(A \cup B) \cap C'$

39. Let

$$A = \{2, 4, 5, 6, 8\}$$
$$B = \{1, 4, 5, 9\}$$
$$C = \{x \mid x \in \mathbb{Z} \text{ and } 2 \le x < 5\}$$

be subsets of $S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Find

★ a. $A \cup B$
★ b. $A \cap B$
★ c. $A \cap C$
d. $B \cup C$
e. $A - B$
f. A'
g. $A \cap A'$
★ h. $(A \cap B)'$
i. $C - B$
j. $(C \cap B) \cup A'$
k. $(B - A)' \cap (A - B)$
l. $(C' \cup B)'$
m. $B \times C$

40. Let

$$A = \{a, \{a\}, \{\{a\}\}\}$$
$$B = \{\varnothing, \{a\}, \{a, \{a\}\}\}$$
$$C = \{a\}$$

be subsets of $S = \{\varnothing, a, \{a\}, \{\{a\}\}, \{a, \{a\}\}\}$. Find

a. $A \cap C$
b. $B \cap C'$
c. $A \cup B$
d. $\varnothing \cap B$
e. $(B \cup C) \cap A$
f. $A' \cap B$
g. $\{\varnothing\} \cap B$

41. Let $S = A \times B$ where $A = \{2, 3, 4\}$ and $B = \{3, 5\}$. Which of the following statements are true?

a. $A \subseteq S$
b. $3 \in S$
c. $(3, 3) \in S$
d. $(5, 4) \in S$
e. $\varnothing \subseteq S$
f. $\{(2, 5)\} \subseteq S$

42. Let

$$A = \{x \mid x \text{ is the name of a former president of the United States}\}$$
$$B = \{\text{Adams, Hamilton, Jefferson, Grant}\}$$
$$C = \{x \mid x \text{ is the name of a state}\}$$

Find

a. $A \cap B$
b. $A \cap C$
c. $B \cap C$

43. Let

$$A = \{x \mid x \text{ is a word that appears before } dog \text{ in an English language dictionary}\}$$
$$B = \{x \mid x \text{ is a word that appears after } canary \text{ in an English language dictionary}\}$$
$$C = \{x \mid x \text{ is a word of more than four letters}\}$$

Which of the following statements are true?

a. $B \subseteq C$
b. $A \cup B = \{x \mid x \text{ is a word in an English language dictionary}\}$
c. $cat \in B \cap C'$
d. $bamboo \in A - B$

★ 44. Consider the following subsets of $\mathbb{Z}$:

$$A = \{x \mid (\exists y)(y \in \mathbb{Z} \text{ and } y \ge 4 \text{ and } x = 3y)\}$$
$$B = \{x \mid (\exists y)(y \in \mathbb{Z} \text{ and } x = 2y)\}$$
$$C = \{x \mid x \in \mathbb{Z} \text{ and } |x| \le 10\}$$

51. For each of the following statements, find general conditions on sets A and B to make the statement true:
    - ★ a. $A \cup B = A$
    - b. $A \cap B = A$
    - c. $A \cup \varnothing = \varnothing$
    - d. $B - A = \varnothing$
    - e. $A \cup B \subseteq A \cap B$
    - f. $A \times B = B \times A$
52. For any finite set S, let $|S|$ denote the number of elements in S. If $|A| = 3$ and $|B| = 4$, find
    - a. $|A \times B|$.
    - b. $|A^2|$.
    - c. $|B^2|$.
    - d. the maximum possible value for $|A \cap B|$.
    - e. the minimum possible value for $|A \cup B|$.
53. Prove that

$$(A \cap B) \subseteq A$$

where A and B are arbitrary sets.

54. Prove that

$$A \subseteq (A \cup B)$$

where A and B are arbitrary sets.

★ 55. Prove that $\wp(A) \cap \wp(B) = \wp(A \cap B)$ where A and B are arbitrary sets.

56. Prove that $\wp(A) \cup \wp(B) \subseteq \wp(A \cup B)$ where A and B are arbitrary sets.
57. Prove that if $A \cup B = A - B$, then $B = \varnothing$. (*Hint:* Do a proof by contradiction.)

★ 58. Prove that if $(A - B) \cup (B - A) = A \cup B$, then $A \cap B = \varnothing$. (*Hint:* Do a proof by contradiction.)

59. Prove that if $C \subseteq B - A$, then $A \cap C = \varnothing$.
60. Prove that $A \subseteq B$ if and only if $A \cap B' = \varnothing$.
61. Prove that $(A \cap B) \cup C = A \cap (B \cup C)$ if and only if $C \subseteq A$.
62. A binary operation on sets called the *symmetric difference* is defined by $A \oplus B = (A - B) \cup (B - A)$.
    - a. Draw a Venn diagram to illustrate $A \oplus B$.
    - b. For $A = \{3, 5, 7, 9\}$ and $B = \{2, 3, 4, 5, 6\}$, what is $A \oplus B$?
    - c. Prove that $A \oplus B = (A \cup B) - (A \cap B)$ for arbitrary sets A and B.
    - d. For an arbitrary set A, what is $A \oplus A$? What is $\varnothing \oplus A$?
    - e. Prove that $A \oplus B = B \oplus A$ for arbitrary sets A and B.
    - f. For any sets A, B, and C, prove that $(A \oplus B) \oplus C = A \oplus (B \oplus C)$.
63. Which of the following are true for all sets A, B, and C?
    - a. $A \cup (B \times C) = (A \cup B) \times (A \cup C)$
    - b. $A \times (B \cap C) = (A \times B) \cap (A \times C)$
    - c. $A \times \varnothing = \varnothing$
    - d. $\wp(A) \times \wp(A) = \wp(A^2)$
    - e. $A \times (B \times C) = (A \times B) \times C$
64. Verify the basic set identities on page 197 by showing set inclusion in each direction. (We have already done 3a and 4a.)
65. A and B are subsets of a set S. Prove the following set identities by showing set inclusion in each direction:
    - a. $(A \cup B)' = A' \cap B'$ } De Morgan's laws
    - b. $(A \cap B)' = A' \cup B'$ } De Morgan's laws
    - ★ c. $A \cup (B \cap A) = A$
    - d. $(A \cap B')' \cup B = A' \cup B$
    - e. $(A \cap B) \cup (A \cap B') = A$
    - f. $[A \cap (B \cup C)]' = A' \cup (B' \cap C')$

EXERCISES 3.2

★ 1. A frozen yogurt shop allows you to choose one flavor (vanilla, strawberry, lemon, cherry, or peach), one topping (chocolate shavings, crushed toffee, or crushed peanut brittle), and one condiment (whipped cream or shredded coconut). How many different desserts are possible?

★ 2. In Exercise 1, how many dessert choices do you have if you are allergic to strawberries and chocolate?

3. A video game on a microcomputer is begun by making selections from each of three menus. The first menu (number of players) has four selections, the second menu (level of play) has eight, and the third menu (speed) has six. In how many configurations can the game be played?

4. A multiple-choice exam has 20 questions, each with four possible answers, and 10 additional questions, each with five possible answers. How many different answer sheets are possible?

5. A user's password to access a computer system consists of three letters followed by two digits. How many different passwords are possible?

6. On the computer system of Exercise 5, how many passwords are possible if uppercase and lowercase letters can be distinguished?

★ 7. A telephone conference call is being placed from Central City to Booneville by way of Cloverdale. There are 45 trunk lines from Central City to Cloverdale and 13 from Cloverdale to Booneville. How many different ways can the call be placed?

8. A, B, C, and D are nodes on a computer network. There are two paths between A and C, two between B and D, three between A and B, and four between C and D. Along how many routes can a message from A to D be sent?

★ 9. How many Social Security numbers are possible?

10. An apartment building purchases a new lock system for its 175 units. A lock is opened by punching in a two-digit code. Has the apartment management made a wise purchase?

★ 11. A palindrome is a string of characters that reads the same forward and backward. How many five-letter English language palindromes are possible?

12. How many three-digit numbers less than 600 can be made using the digits 8, 6, 4, and 2?

13. A binary logical connective can be defined by giving its truth table. How many different binary logical connectives are there?

Exercises 14–17 are related to Example 29.

★ 14. Show that the 4 juggling patterns of length 2 using at most 2 balls—(3, 1), (1, 3), (2, 2), and (1, 1)—have stack numbers of 21, 12, 22, and 11, respectively.

15. a. Using 3 balls, find the juggling pattern of length 2 shown in the table.

G	G	B	B	R	R	G	G	B	B	R	R	...

b. Find the stack number for this juggling pattern.

c. Given a stack number of 221, draw a table for the corresponding juggling pattern of length 3 using 2 balls.

16. a. What is the number of juggling patterns of length 2 using at most 3 balls?

b. Write the stack numbers for the patterns of part (a).

c. Write the tables for these juggling patterns.

17. What is the number of juggling patterns of length 3 using at most 4 balls?

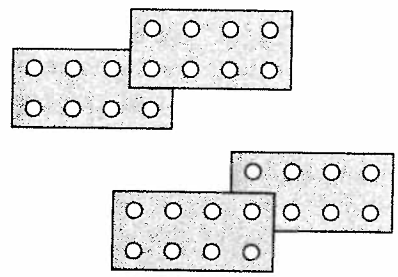
18. In the original BASIC programming language, an identifier must be either a single letter or a letter followed by a single digit. How many identifiers are possible?
19. Three seats on the county council are to be filled, each with someone from a different party. There are four candidates running from the Concerned Environmentalist party, three from the Limited Development party, and two from the Friends of the Spotted Newt party. In how many ways can the seats be filled?
20. A president and vice-president must be chosen for the executive committee of an organization. There are 17 volunteers from the Eastern Division and 24 volunteers from the Western Division. If both officers must come from the same division, in how many ways can the officers be selected?
21. A dinner special allows you to select from five appetizers, three salads, four entrees, and three beverages. How many different dinners are there?
22. In Exercise 21, how many different dinners are there if you may have an appetizer or a salad but not both?
23. A new car can be ordered with a choice of 10 exterior colors; 7 interior colors; automatic, 3-speed, or 5-speed transmission; with or without air conditioning; with or without power steering; and with or without the option package that contains the power door lock and the rear window defroster. How many different cars can be ordered?
24. In Exercise 23, how many different cars can be ordered if the option package comes only on a car with an automatic transmission?
25. In one state, automobile license plates must have two digits (no leading zeros) followed by one letter followed by a string of two to four digits (leading zeros are allowed). How many different plates are possible?
26. A Hawaiian favorite fast food is the "loco moco," invented at a Hilo restaurant. It consists of a bed of rice under a meat patty with egg on top, the whole thing smothered in brown gravy. The rice can be white rice or brown rice, the egg can be fried, scrambled, or poached, and the meat can be hamburger, Spam, Portuguese sausage, bacon, turkey, hot dog, salmon, or mahi. How many different loco mocos can be ordered?
27. A customer at a fast-food restaurant can order a hamburger with or without mustard, ketchup, pickle, or onion; a fish sandwich with or without lettuce, tomato, or tartar sauce; and a choice of three kinds of soft drinks or two kinds of milk shakes. How many different orders are possible if a customer can order at most one hamburger, one fish sandwich, and one beverage but can order less?
28. What is the value of *Count* after the following pseudocode has been executed?

```
Count = 0
for i = 1 to 5 do
   for Letter = 'A' to 'C' do
      Count = Count + 1
   end for
end for
```

29. What is the value of *Result* after the following pseudocode has been executed?

```
Result = 0
for Index = 20 down to 10 do
   for Inner = 5 to 10 do
      Result = Result + 2
   end for
end for
```

30. How many unique ways are there to stack two 2 × 4 Lego bricks of the same color? Two stacks that look the same if you merely rotate them are considered to be the same arrangement. Here is one such stack, the original on top and the same stack after rotating 180°.



Exercises 31–36 concern the set of three-digit integers (numbers between 100 and 999 inclusive).

31. How many are divisible by 5?
★ 32. How many are not divisible by 5?
33. How many are divisible by 4?
34. How many are divisible by 4 or 5?
35. How many are divisible by 4 and 5?
36. How many are divisible by neither 4 nor 5?

Exercises 37–46 concern the set of binary strings of length 8 (each character is either the digit 0 or the digit 1).

★ 37. How many such strings are there?
38. How many begin and end with 0?
★ 39. How many begin or end with 0?
40. How many have 1 as the second digit?
41. How many begin with 111?
42. How many contain exactly one 0?
43. How many begin with 10 or have a 0 as the third digit?
44. How many are palindromes? (See Exercise 11.)
★ 45. How many contain exactly seven 1s?
46. How many contain two or more 0s?

In Exercises 47–51, two dice are rolled, one black and one white.

47. How many different rolls are possible? (Note that a 4-black, 1-white result and a 1-black, 4-white result are two different outcomes.)
★ 48. How many rolls result in doubles (both dice showing the same value)?
49. How many rolls result in "snake eyes" (both dice showing 1)?
50. How many rolls result in a total of 7 or 11?
51. How many rolls occur in which neither die shows the value 4?

In Exercises 52–56, a customer is ordering a computer. The choices are 17", 19", 21", or 23" monitor; 1.0 GHz, 1.3 GHz, 1.5 GHz, 1.7 GHz, or 2.0 GHz processor; 12×, 14×, or

24× CD drive; 128 MB, 256 MB, or 512 MB of RAM; optional fax card; optional sound card.

52. How many different machine configurations are possible?

★ 53. How many different machines can be ordered with a 1.7 GHz processor?

54. How many different machines can be ordered with a 21" monitor but no sound card and no fax card?

55. How many different machines can be ordered with no monitor?

56. How many different machines can be ordered with a minimum 1.5 GHz processor and either a sound card or a fax card but not both?

In Exercises 57–66, a hand consists of 1 card drawn from a standard 52-card deck with flowers on the back and 1 card drawn from a standard 52-card deck with birds on the back. A standard deck has 13 cards from each of 4 suits (clubs, diamonds, hearts, spades). The 13 cards have face value 2 through 10, jack, queen, king, or ace. Each face value is a "kind" of card. The jack, queen, and king are "face cards."

★ 57. How many different hands are possible? (Note that a flower-ace-of-spades, bird-queen-of-hearts and a flower-queen-of-hearts, bird-ace-of-spades are two different outcomes.)

58. How many hands consist of a pair of aces?

59. How many hands contain all face cards?

★ 60. How many hands consist of two of a kind? (two aces, two jacks, etc.)

61. How many hands contain exactly one king?

62. How many hands have a face value of 5 (aces count as 1, face cards count as 10)?

63. How many hands have a face value of less than 5 (aces count as 1, face cards count as 10)?

64. How many hands do not contain any face cards?

★ 65. How many hands contain at least one face card?

66. How many hands contain at least one king?

Exercises 67–70 are related to Example 38.

67. Translate the IP address 56.201.33.10 into its binary form.

68. What part of the binary string from Exercise 67 is the netid of this address?

★ 69. Compute the total number of IP addresses available under IPv4 taking the following restrictions into account: class A networks cannot have netids of all 1s, and no host id can be all 0s or all 1s.

70. In 1994, there were about 1.3 million host machines connected to the Internet. Assume that the number doubles each year. Write and solve a recurrence relation for the number of hosts connected to the Internet. Using your answer from Exercise 69, when might the number of IP addresses be exhausted?

71. Draw a decision tree to find the number of binary strings of length 4 that do not have consecutive 0s. (Compare your answer with the one for Exercise 35 of Section 2.5.)

★ 72. Voting on a certain issue is conducted by having everyone put a red, blue, or green slip of paper into a hat. Then the slips are pulled out one at a time. The first color to receive two votes wins. Draw a decision tree to find the number of ways in which the balloting can occur.

73. Draw a decision tree (use teams A and B) to find the number of ways the NBA playoffs can happen, where the winner is the first team to win 4 out of 7.