6. Justify each step in the following proof sequence of

$$(\exists x)[P(x) \to Q(x)] \to [(\forall x)P(x) \to (\exists x)Q(x)]$$

1. $(\exists x)[P(x) \to Q(x)]$
2. $P(a) \to Q(a)$
3. $(\forall x)P(x)$
4. $P(a)$
5. $Q(a)$
6. $(\exists x)Q(x)$

7. Justify each step in the following proof sequence of

$$(\exists x)P(x) \wedge (\forall x)(P(x) \to Q(x)) \to (\exists x)Q(x)$$

1. $(\exists x)P(x)$
2. $(\forall x)(P(x) \to Q(x))$
3. $P(a)$
4. $P(a) \to Q(a)$
5. $Q(a)$
6. $(\exists x)Q(x)$

★ 8. Consider the wff $(\forall x)[(\exists y)P(x, y) \wedge (\exists y)Q(x, y)] \to (\forall x)(\exists y)[P(x, y) \wedge Q(x, y)]$.
   a. Find an interpretation to prove that this wff is not valid.
   b. Find the flaw in the following "proof" of this wff.

1. $(\forall x)[(\exists y)P(x, y) \wedge (\exists y)Q(x, y)]$	hyp
2. $(\forall x)[P(x, a) \wedge Q(x, a)]$	1, ei
3. $(\forall x)(\exists y)[P(x, y) \wedge Q(x, y)]$	2, eg

9. Consider the wff $(\forall y)(\exists x)Q(x, y) \to (\exists x)(\forall y)Q(x, y)$.
   a. Find an interpretation to prove that this wff is not valid.
   b. Find the flaw in the following "proof" of this wff.

1. $(\forall y)(\exists x)Q(x, y)$	hyp
2. $(\exists x)Q(x, y)$	1, ui
3. $Q(a, y)$	2, ei
4. $(\forall y)Q(a, y)$	3, ug
5. $(\exists x)(\forall y)Q(x, y)$	4, eg

In Exercises 10–14, prove that each wff is a valid argument.

★ 10. $(\forall x)P(x) \to (\forall x)[P(x) \vee Q(x)]$
11. $(\forall x)P(x) \wedge (\exists x)Q(x) \to (\exists x)[P(x) \wedge Q(x)]$
12. $(\exists x)(\exists y)P(x, y) \to (\exists y)(\exists x)P(x, y)$
13. $(\forall x)(\forall y)Q(x, y) \to (\forall y)(\forall x)Q(x, y)$
14. $(\forall x)P(x) \wedge (\exists x)[P(x)]' \to (\exists x)Q(x)$

In Exercises 15–27, either prove that the wff is a valid argument or give an interpretation in which it is false.

★ 15. $(\exists x)[A(x) \wedge B(x)] \to (\exists x)A(x) \wedge (\exists x)B(x)$
16. $(\exists x)[R(x) \vee S(x)] \to (\exists x)R(x) \vee (\exists x)S(x)$

17. $(\forall x)[P(x) \to Q(x)] \to [(\forall x)P(x) \to (\forall x)Q(x)]$
18. $(\forall x)(P(x))' \to (\forall x)(P(x) \to Q(x))$
19. $[(\forall x)P(x) \to (\forall x)Q(x)] \to (\forall x)[P(x) \to Q(x)]$
★ 20. $(\exists x)(\forall y Q(x, y) \to (\forall y)(\exists x)Q(x, y)$
21. $(\forall x)P(x) \lor (\exists x)Q(x) \to (\forall x)[P(x) \lor Q(x)]$
22. $(\forall x)[A(x) \to B(x)] \to [(\exists x)A(x) \to (\exists x)B(x)]$
23. $(\forall y)[Q(x, y) \to P(x)] \to [(\exists y)Q(x, y) \to P(x)]$
★ 24. $[P(x) \to (\exists y)Q(x, y)] \to (\exists y)[P(x) \to Q(x, y)]$
25. $(\forall x)(P(x) \lor Q(x)) \land (\exists x)Q(x) \to (\exists x)P(x))$
26. $(\exists x)[P(x) \to Q(x)] \land (\forall y)[Q(y) \to R(y)] \land (\forall x)P(x) \to (\exists x)R(x)$
27. $(\forall x)(\forall y)[(P(x) \land S(x, y)) \to Q(y)] \land (\exists x)B(x) \land (\forall x)(B(x) \to P(x)) \land (\forall x)(\exists y)S(x, y) \to (\exists x)Q(x)$

28. The Greek philosopher Aristotle (384–322 B.C.) studied under Plato and tutored Alexander the Great. His studies of logic influenced philosophers for hundreds of years. His four "perfect" syllogisms are identified by the names given them by medieval scholars. For each, formulate the argument in predicate logic notation and then provide a proof.
    a. "Barbara"
       All *M* are *P*.
       All *S* are *M*.
       Therefore all *S* are *P*.
    b. "Celarent"
       No *M* are *P*.
       All *S* are *M*.
       Therefore no *S* are *P*.
    c. "Darii"
       All *M* are *P*.
       Some *S* are *M*.
       Therefore some *S* are *P*.
    d. "Ferio"
       No *M* are *P*.
       Some *S* are *M*.
       Therefore some *S* are not *P*.

Using predicate logic, prove that each argument in Exercises 29–37 is valid. Use the predicate symbols shown.

29. Some plants are flowers. All flowers smell sweet. Therefore, some plants smell sweet. $P(x)$, $F(x)$, $S(x)$
★ 30. Every crocodile is bigger than every alligator. Sam is a crocodile. But there is a snake, and Sam isn't bigger than that snake. Therefore, something is not an alligator. $C(x)$, $A(x)$, $B(x, y)$, s, $S(x)$
31. There is an astronomer who is not nearsighted. Everyone who wears glasses is nearsighted. Furthermore, everyone either wears glasses or wears contact lenses. Therefore, some astronomer wears contact lenses. $A(x)$, $N(x)$, $G(x)$, $C(x)$
★ 32. Every member of the board comes from industry or government. Everyone from government who has a law degree is in favor of the motion. John is not from industry, but he does have a law degree. Therefore, if John is a member of the board, he is in favor of the motion. $M(x)$, $I(x)$, $G(x)$, $L(x)$, $F(x)$, j

33. There is some movie star who is richer than everyone. Anyone who is richer than anyone else pays more taxes than anyone else does. Therefore, there is a movie star who pays more taxes than anyone. $M(x), R(x, y), T(x, y)$
34. Every computer science student works harder than somebody, and everyone who works harder than any other person gets less sleep than that person. Maria is a computer science student. Therefore, Maria gets less sleep than someone else. $C(x), W(x, y), S(x, y), m$
35. Every ambassador speaks only to diplomats, and some ambassador speaks to someone. Therefore, there is a diplomat. $A(x), S(x, y), D(x)$
36. Some elephants are afraid of all mice. Some mice are small. Therefore there is an elephant that is afraid of something small. $E(x), M(x), A(x, y), S(x)$
37. Every farmer owns a cow. No dentist owns a cow. Therefore no dentist is a farmer. $F(x), C(x), O(x, y), D(x)$

38. Prove that

$$[(\forall x)P(x)]' \leftrightarrow (\exists x)[P(x)]'$$

is valid. (*Hint:* Instead of a proof sequence, use Example 32 and substitute equivalent expressions.)

39. The equivalence of Exercise 38 says that if it is false that every element of the domain has property P, then some element of the domain fails to have property P, and vice versa. The element that fails to have property P is called a counterexample to the assertion that every element has property P. Thus a counterexample to the assertion

$$(\forall x)(x \text{ is odd})$$

in the domain of integers is the number 10, an even integer. (Of course, there are lots of other counterexamples to this assertion.) Find counterexamples in the domain of integers to the following assertions. (An integer $x > 1$ is prime if the only factors of x are 1 and x.)

a. $(\forall x)(x \text{ is negative})$
b. $(\forall x)(x \text{ is the sum of even integers})$
c. $(\forall x)(x \text{ is prime} \rightarrow x \text{ is odd})$
d. $(\forall x)(x \text{ prime} \rightarrow (-1)^x = -1)$
e. $(\forall x)(x \text{ prime} \rightarrow 2^x - 1 \text{ is prime})$

SECTION 1.5 Logic Programming

The programming languages with which you are probably familiar, such as C++ and Java, are known as **procedural languages.** Much of the content of a program written in a procedural language consists of instructions to carry out the algorithm the programmer believes will solve the problem at hand. The programmer, therefore, is telling the computer how to solve the problem in a step-by-step fashion.

Some programming languages, rather than being procedural, are **declarative languages** or **descriptive languages.** A declarative language is based on predicate logic; such a language comes equipped with its own rules of inference. A program written in a declarative language consists only of statements—actually predicate wffs—that are declared as hypotheses. Execution of a declarative program allows the user to pose queries, asking for information about