HW2, 2510 Fall 2007

3. Give the truth value of each of the following wffs in the interpretation where the domain consists of the states of the United States, $Q(x, y)$ is "x is north of y," $P(x)$ is "x starts with the letter M," and a is "Massachusetts."
   a. $(\forall x)P(x)$
   b. $(\forall x)(\forall y)(\forall z)[Q(x, y) \wedge Q(y, z) \to Q(x, z)]$
   c. $(\exists y)(\exists x)Q(y, x)$
   d. $(\forall x)(\exists y)[P(y) \wedge Q(x, y)]$
   e. $(\exists y)Q(a, y)$
   f. $(\exists x)[P(x) \wedge Q(x, a)]$
4. Give the truth value of each of the following wffs in the interpretation where the domain consists of people: $M(x, y)$ is "x is the mother of y," $F(x)$ is "x is female," $M(x)$ is "m is male."
   a. $(\forall x)(\exists y)(M(y, x))$
   b. $(\exists x)(\forall y)(M(x, y))$
   c. $(\forall x)(\forall y)(M(x, y) \to M(y))$
   d. $(\exists x)(\exists y)(M(x, y) \wedge M(y))$
   e. $(\exists x)(\forall y)(M(x, y) \to F(y))$
5. For each wff, find an interpretation in which it is true and one in which it is false.
   ★ a. $(\forall x)([A(x) \vee B(x)] \wedge [A(x) \wedge B(x)]')$
   b. $(\forall x)(\forall y)[P(x, y) \to P(y, x)]$
   c. $(\forall x)[P(x) \to (\exists y)Q(x, y)]$
   d. $(\exists x)[A(x) \wedge (\forall y)B(x, y)]$
   e. $[(\forall x)A(x) \to (\forall x)B(x)] \to (\forall x)[A(x) \to B(x)]$
6. Identify the scope of each of the quantifiers in the following wffs and indicate any free variables.
   a. $(\forall x)[P(x) \to Q(y)]$
   b. $(\exists x)[A(x) \wedge (\forall y)B(y)]$
   c. $(\exists x)[(\forall y)P(x, y) \wedge Q(x, y)]$
   d. $(\exists x)(\exists y)[A(x, y) \wedge B(y, z) \to A(a, z)]$
7. Which of the following are equivalent to the statement

   All circles are round.

   a. If it's round, it's a circle.
   b. Roundness is a necessary property of circles.
   c. Something that isn't round can't be a circle.
   d. Some round things are circles.
8. Which of the following are equivalent to the statement

   Cats are smarter than dogs.

   a. Some cats are smarter than some dogs.
   b. There is a cat that is smarter than all dogs.
   c. All cats are smarter than all dogs.
   d. Only cats are smarter than dogs.
   e. All cats are smarter than any dog.
9. Using the predicate symbols shown and appropriate quantifiers, write each English language statement as a predicate wff. (The domain is the whole world.)

   $D(x)$ is "x is a day."
   $S(x)$ is "x is sunny."
   $R(x)$ is "x is rainy."
   M is "Monday."
   T is "Tuesday."

a. All days are sunny.
b. Some days are not rainy.
c. Every day that is sunny is not rainy.
d. Some days are sunny and rainy.
e. No day is both sunny and rainy.
f. It is always a sunny day only if it is a rainy day.
g. No day is sunny.
h. Monday was sunny; therefore every day will be sunny.
i. It rained both Monday and Tuesday.
j. If some day is rainy, then every day will be sunny.

10. Using the predicate symbols shown and appropriate quantifiers, write each English language statement as a predicate wff. (The domain is the whole world.)

$B(x)$ is "x is a ball."
$R(x)$ is "x is round."
$S(x)$ is "x is a soccer ball."

a. All balls are round.
b. Not all balls are soccer balls.
c. All soccer balls are round.
d. Some balls are not round.
e. Some balls are round but soccer balls are not.
f. Every round ball is a soccer ball.
g. Only soccer balls are round balls.
h. If soccer balls are round, then all balls are round.

11. Using the predicate symbols shown and appropriate quantifiers, write each English language statement as a predicate wff. (The domain is the whole world.)

$P(x)$ is "x is a person."
$T(x)$ is "x is a time."
$F(x, y)$ is "x is fooled at y."

a. You can fool some of the people all of the time.
b. You can fool all of the people some of the time.
c. You can't fool all of the people all of the time.

12. Using the predicate symbols shown and appropriate quantifiers, write each English language statement as a predicate wff. (The domain is the whole world.)

$L(x)$: x is a lion.
$R(x)$: x roars.
$P(x)$: x is a predator.
$Z(x)$: x is a zebra.
$E(x, y)$: x eats y.

a. All lions are predators.
b. Some lions roar.
c. Only lions roar.
d. Some lions eat all zebras.
e. All lions eat all zebras.

13. Using the predicate symbols shown and appropriate quantifiers, write each English language statement as a predicate wff. (The domain is the whole world.)

    $G(x)$: x is a game.
    $M(x)$: is a movie.
    $F(x, y)$: x is more fun than y.

    a. Any movie is more fun than any game.
    b. No game is more fun than every movie.
    c. Only games are more fun than movies.
    d. All games are more fun than some movie.

14. Using the predicate symbols shown and appropriate quantifiers, write each English language statement as a predicate wff. (The domain is the whole world.)

    $J(x)$ is "x is a judge."
    $C(x)$ is "x is a chemist."
    $L(x)$ is "x is a lawyer."
    $W(x)$ is "x is a woman."
    $A(x, y)$ is "x admires y."

    a. There are some women lawyers who are chemists.
    ★ b. No woman is both a lawyer and a chemist.
    ★ c. Some lawyers admire only judges.
    d. All judges admire only judges.
    e. Only judges admire judges.
    f. All women lawyers admire some judge.
    g. Some women admire no lawyer.

15. Using the predicate symbols shown and appropriate quantifiers, write each English language statement as a predicate wff. (The domain is the whole world.)

    $C(x)$ is "x is a Corvette."
    $P(x)$ is "x is a Porsche."
    $F(x)$ is "x is a Ferrari."
    $S(x, y)$ is "x is slower than y."

    ★ a. Nothing is both a Corvette and a Ferrari.
    ★ b. Some Porsches are slower than only Ferraris.
    c. Only Corvettes are slower than Porsches.
    d. All Ferraris are slower than some Corvette.
    e. Some Porsches are slower than no Corvette.
    f. If there is a Corvette that is slower than a Ferrari, then all Corvettes are slower than all Ferraris.

16. Using the predicate symbols shown and appropriate quantifiers, write each English language statement as a predicate wff. (The domain is the whole world.)

19. Three forms of negation are given for each statement. Which is correct?
    - ★ a. Some people like mathematics.
        1. Some people dislike mathematics.
        2. Everybody dislikes mathematics.
        3. Everybody likes mathematics.
    - b. Everyone loves ice cream.
        1. No one loves ice cream.
        2. Everyone dislikes ice cream.
        3. Someone doesn't love ice cream.
    - c. All people are tall and thin.
        1. Someone is short and fat.
        2. No one is tall and thin.
        3. Someone is short or fat.
    - d. Some pictures are old or faded.
        1. Every picture is neither old nor faded.
        2. Some pictures are not old or faded.
        3. All pictures are not old or not faded.
20. Three forms of negation are given for each statement. Which is correct?
    - a. Nobody is perfect.
        1. Everyone is imperfect.
        2. Everyone is perfect.
        3. Someone is perfect.
    - b. All swimmers are tall.
        1. Some swimmer is not tall.
        2. There are no tall swimmers.
        3. Every swimmer is short.
    - c. Every planet is cold and lifeless.
        1. No planet is cold and lifeless.
        2. Some planet is not cold and not lifeless.
        3. Some planet is not cold or not lifeless.
    - d. No bears are hungry.
        1. Only bears are hungy.
        2. All bears are hungry.
        3. There is a hungry bear.
21. Write the negation of each of the following.
    - a. Some Web sites feature audio.
    - b. Every Web site has both audio and video.
    - ★ c. Every Web site has either audio or video.
    - d. Some Web sites have neither audio nor video.
    - e. Every Web site either has text or else has both audio and video.
22. Write the negation of each of the following.
    - a. Only students eat pizza.
    - b. Every student eats pizza
    - c. Some students eat only pizza.
23. Write the negation of each of the following.
    - a. Some farmer grows only corn.
    - b. All farmers grow corn.
    - c. Corn is grown only by farmers.

24. Explain why each wff is valid.
    a. $(\forall x)(\forall y)A(x, y) \leftrightarrow (\forall y)(\forall x)A(x, y)$
    b. $(\exists x)(\exists y)A(x, y) \leftrightarrow (\exists y)(\exists x)A(x, y)$
    ★ c. $(\exists x)(\forall y)P(x, y) \rightarrow (\forall y)(\exists x)P(x, y)$
    d. $A(a) \rightarrow (\exists x)A(x)$
    e. $(\forall x)[A(x) \rightarrow B(x)] \rightarrow [(\forall x)A(x) \rightarrow (\forall x)B(x)]$
25. Give interpretations to prove that each of the following wffs is not valid:
    ★ a. $(\exists x)A(x) \wedge (\exists x)B(x) \rightarrow (\exists x)[A(x) \wedge B(x)]$
    b. $(\forall x)(\exists y)P(x, y) \rightarrow (\exists x)(\forall y)P(x, y)$
    c. $(\forall x)[P(x) \rightarrow Q(x)] \rightarrow [(\exists x)P(x) \rightarrow (\forall x)Q(x)]$
    d. $(\forall x)[A(x)]' \leftrightarrow [(\forall x)A(x)]'$
26. Decide whether each of the following wffs is valid or invalid. Justify your answer.
    a. $(\exists x)A(x) \leftrightarrow ((\forall x)[A(x)]')'$
    b. $(\forall x)P(x) \vee (\exists x)Q(x) \rightarrow (\forall x)[P(x) \vee Q(x)]$
    c. $(\forall x)A(x) \rightarrow ((\exists x)[A(x)]')'$
    d. $(\forall x)[P(x) \vee Q(x)] \rightarrow (\forall x)P(x) \vee (\exists y)Q(y)$

SECTION 1.4 Predicate Logic

We can imagine arguments of the form

$$P_1 \wedge P_2 \wedge P_3 \wedge \cdots \wedge P_n \rightarrow Q$$

where the wffs are built from predicates and quantifiers as well as logical connectives and grouping symbols. For a **valid argument**, Q must follow logically from $P_1, \ldots, P_n$ based solely on the internal structure of the argument, not on the truth or falsity of Q in any particular interpretation. In other words, the wff

$$P_1 \wedge P_2 \wedge P_3 \wedge \cdots \wedge P_n \rightarrow Q$$

must be valid—true in all possible interpretations. No equivalent of the truth table exists to easily prove validity, so we turn to a formal logic system called **predicate logic.** We again use a system of derivation rules to build a proof sequence leading from the hypotheses to the conclusion. The rules should once more be truth-preserving so that if in some interpretation all the hypotheses are true, then the conclusion will also be true in that interpretation. The system will then be correct (*only* valid arguments will be provable). We also want the system to be complete (*every* valid argument should be provable), yet at the same time the rule set should be minimal.

Derivation Rules for Predicate Logic

The equivalence rules and inference rules of propositional logic are still part of predicate logic. An argument of the form

$$P \wedge (P \rightarrow Q) \rightarrow Q$$

is still valid by modus ponens, even if the wffs involved are predicate wffs.