EXERCISES 1.1

Answers to starred items are given in the back of the book.

★ 1. Which of the following are statements?
   a. The moon is made of green cheese.
   b. He is certainly a tall man.
   c. Two is a prime number.
   d. Will the game be over soon?
   e. Next year interest rates will rise.
   f. Next year interest rates will fall.
   g. $x^2 - 4 = 0$

2. Given the truth values A true, B false, and C true, what is the truth value of each of the following wffs?
   a. $A \wedge (B \vee C)$
   b. $(A \wedge B) \vee C$
   c. $(A \wedge B)' \vee C$
   d. $A' \vee (B' \wedge C)'$

3. What is the truth value of each of the following statements?
   a. 8 is even or 6 is odd.
   b. 8 is even and 6 is odd.
   c. 8 is odd or 6 is odd.
   d. 8 is odd and 6 is odd.
   e. If 8 is odd, then 6 is odd.
   f. If 8 is even, then 6 is odd.
   g. If 8 is odd, then 6 is even.
   h. If 8 is odd and 6 is even, then $8 < 6$.

★ 4. Find the antecedent and consequent in each of the following statements.
   a. Healthy plant growth follows from sufficient water.
   b. Increased availability of information is a necessary condition for further technological advances.
   c. Errors will be introduced only if there is a modification of the program.
   d. Fuel savings implies good insulation or storm windows throughout.

5. Several forms of negation are given for each of the following statements. Which are correct?
   a. The answer is either 2 or 3.
      1. Neither 2 nor 3 is the answer.
      2. The answer is not 2 or not 3.
      3. The answer is not 2 and it is not 3.
   b. Cucumbers are green and seedy.
      1. Cucumbers are not green and not seedy.
      2. Cucumbers are not green or not seedy.
      3. Cucumbers are green and not seedy.
   c. $2 < 7$ and 3 is odd.
      1. $2 > 7$ and 3 is even.
      2. $2 \geq 7$ and 3 is even.
      3. $2 \geq 7$ or 3 is odd.
      4. $2 \geq 7$ or 3 is even.

6. Several forms of negation are given for each statement. Which are correct?
   a. The carton is sealed or the milk is sour.
      1. The milk is not sour or the carton is not sealed.
      2. The carton is not sealed and also the milk is not sour.
      3. If the carton is not sealed, then the milk will be sour.

b. Flowers will bloom only if it rains.
   1. The flowers will bloom but it will not rain.
   2. The flowers will not bloom and it will not rain.
   3. The flowers will not bloom or else it will not rain.

c. If you build it, they will come.
   1. If you build it, then they won't come.
   2. You don't build it, but they do come.
   3. You build it, but they don't come.

7. Write the negation of each statement.

★ a. If the food is good, then the service is excellent.

★ b. Either the food is good or the service is excellent.

c. Either the food is good and the service is excellent, or else the price is high.

d. Neither the food is good nor the service excellent.

e. If the price is high, then the food is good and the service is excellent.

8. Write the negation of each statement.

a. The processor is fast but the printer is slow.

b. The processor is fast or else the printer is slow.

c. If the processor is fast, then the printer is slow.

d. Either the processor is fast and the printer is slow, or else the file is damaged.

e. If the file is not damaged and the processor is fast, then the printer is slow.

f. The printer is slow only if the file is damaged.

9. Let A, B, and C be the following statements:

A Roses are red.
B Violets are blue.
C Sugar is sweet.

Translate the following compound statements into symbolic notation.

a. Roses are red and violets are blue.

b. Roses are red, and either violets are blue or sugar is sweet.

c. Whenever violets are blue, roses are red and sugar is sweet.

d. Roses are red only if violets aren't blue or sugar is sour.

e. Roses are red and, if sugar is sour, then either violets aren't blue or sugar is sweet.

10. Let A, B, C, and D be the following statements:

A The villain is French.
B The hero is American.
C The heroine is British.
D The movie is good.

Translate the following compound statements into symbolic notation.

a. The hero is American and the movie is good.

b. Although the villain is French, the movie is good.

c. If the movie is good, then either the hero is American or the heroine is British.

d. The hero is not American, but the villain is French.

e. A British heroine is a necessary condition for the movie to be good.

11. Use A, B, and C as defined in Exercise 9 to translate the following statements into English.
    a. $B \vee C'$
    b. $B' \vee (A \rightarrow C)$
    c. $(C \wedge A') \leftrightarrow B$
    d. $C \wedge (A' \leftrightarrow B)$
    e. $(B \wedge C')' \rightarrow A$
    f. $A \vee (B \wedge C')$
    g. $(A \vee B) \wedge C'$

★ 12. Using letters for the component statements, translate the following compound statements into symbolic notation.
    a. If prices go up, then housing will be plentiful and expensive; but if housing is not expensive, then it will still be plentiful.
    b. Either going to bed or going swimming is a sufficient condition for changing clothes; however, changing clothes does not mean going swimming.
    c. Either it will rain or it will snow but not both.
    d. If Janet wins or if she loses, she will be tired.
    e. Either Janet will win or, if she loses, she will be tired.

13. Using letters H, K, A for the component statements, translate the following compound statements into symbolic notation.
    a. If the horse is fresh, then the knight will win.
    b. The knight will win only if the horse is fresh and the armor is strong.
    c. A fresh horse is a necessary condition for the knight to win.
    d. The knight will win if and only if the armor is strong.
    e. A sufficient condition for the knight to win is that the armor is strong or the horse is fresh.

14. Using letters A, T, E for the component statements, translate the following compound statements into symbolic notation.
    a. If Anita wins the election, then tax rates will be reduced.
    b. Tax rates will be reduced only if Anita wins the election and the economy remains strong.
    c. Tax rates will be reduced if the economy remains strong.
    d. A strong economy will follow from Anita winning the election.
    e. The economy will remain strong if and only if Anita wins the election or tax rates are reduced.

15. Using letters F, B, S for the component statements, translate the following compound statements into symbolic notation.
    a. Plentiful fish are a sufficient condition for bears to be happy.
    b. Bears are happy only if there are plentiful fish.
    c. Unhappy bears means that the fish are not plentiful and also that there is heavy snow.
    d. Unhappy bears is a necessary condition for heavy snow.
    e. The snow is heavy if and only if the fish are not plentiful.

16. Using letters P, C, B, L for the component statements, translate the following compound statements into symbolic notation.
    a. If the project is finished soon, then the client will be happy and the bills will be paid.
    b. If the bills are not paid, then the lights will go out.
    c. The project will be finished soon only if the lights do not go out.
    d. If the bills are not paid and the lights go out, then the client will not be happy.
    e. The bills will be paid if and only if the project is finished soon, or else the lights go out.
    f. The bills will be paid if and only if either the project is finished soon or the lights go out.

17. Construct truth tables for the following wffs. Note any tautologies or contradictions.
    * a. $(A \rightarrow B) \leftrightarrow A' \vee B$
    * b. $(A \wedge B) \vee C \rightarrow A \wedge (B \vee C)$
    c. $A \wedge (A' \vee B')'$
    d. $A \wedge B \rightarrow A'$
    e. $(A \rightarrow B) \rightarrow [(A \vee C) \rightarrow (B \vee C)]$
    f. $A \rightarrow (B \rightarrow A)$
    g. $A \wedge B \leftrightarrow B' \vee A'$
    h. $(A \vee B') \wedge (A \wedge B)'$
    i. $[(A \vee B) \wedge C'] \rightarrow A' \vee C$
* 18. A memory chip from a microcomputer has 2^4 bistable (ON–OFF) memory elements. What is the total number of ON–OFF configurations?
19. Verify the equivalences in the list on page 8 by constructing truth tables. (We have already verified 1a, 4b, and 5a.)
20. Verify by constructing truth tables that the following wffs are tautologies.
    * a. $A \vee A'$
    b. $(A')' \leftrightarrow A$
    * c. $A \wedge B \rightarrow B$
    d. $A \rightarrow A \vee B$
    e. $(A \vee B)' \leftrightarrow A' \wedge B'$ (De Morgan's law)
    f. $(A \wedge B)' \leftrightarrow A' \vee B'$ (De Morgan's law)
    g. $A \vee A \leftrightarrow A$
21. Prove the following tautologies by starting with the left side and finding a series of equivalent wffs that will convert the left side into the right side. You may use any of the equivalencies in the list on page 8 or in Exercise 20.
    a. $(A \wedge B') \wedge C \leftrightarrow (A \wedge C) \wedge B'$
    b. $(A \vee B) \wedge (A \vee B') \leftrightarrow A$
    c. $A \vee (B \wedge A') \leftrightarrow A \vee B$
    d. $(A \wedge B')' \vee B \leftrightarrow A' \vee B$
    e. $A \wedge (A \wedge B')' \leftrightarrow A \wedge B$
* 22. Write a logical expression for a Web search engine to find sites pertaining to dogs that are not retrievers.
23. Write a logical expression for a Web search engine to find sites pertaining to oil paintings by Van Gogh or Rembrandt but not Vermeer.
24. Write a logical expression for a Web search engine to find sites pertaining to novels or plays about AIDS.
25. Consider the following pseudocode.

```
repeat
    i = 1
    read a value for x
    if ((x < 5.0) and (2x < 10.7)) or (sqrt(5x) > 5.1) then
        write the value of x
    end if
    increase i by 1
until i > 5
```

    The input values for x are 1.0, 5.1, 2.4, 7.2, and 5.3. What are the output values?
26. Suppose that A, B, and C represent conditions that will be true or false when a certain computer program is executed. Suppose further that you want the program to carry out a certain task only when A or B is true (but not both) and C is false. Using A, B, and C and the connectives AND, OR, and NOT, write a statement that will be true only under these conditions.

★ 35. The binary connective $|$ is defined by the following truth table:

A	B	$A\|B$
T	T	F
T	F	T
F	T	T
F	F	T

Show that every compound wff is equivalent to a wff using only the connective $|$. (*Hint:* Use Exercise 32 and find equivalent statements for $A \wedge B$ and A' in terms of $|$.)

36. The binary connective $\downarrow$ is defined by the following truth table:

A	B	$A\downarrow B$
T	T	F
T	F	F
F	T	F
F	F	T

Show that every compound statement is equivalent to a statement using only the connective $\downarrow$. (*Hint:* See Exercise 35.)

37. Propositional wffs and truth tables belong to a system of *two-valued logic* because everything has one of two values, false or true. *Three-valued logic* allows a third value of null or "unknown" (Section 4.3 discusses the implications of three-valued logic on databases). The truth tables for this three-valued system follow.

A	B	$A \wedge B$
T	T	T
T	F	F
T	N	N
F	T	F
F	F	F
F	N	F
N	T	N
N	F	F
N	N	N

A	B	$A \vee B$
T	T	T
T	F	T
T	N	T
F	T	T
F	F	F
F	N	N
N	T	T
N	F	N
N	N	N

A	A'
T	F
F	T
N	N

a. Viewing N as "unknown," explain why it is reasonable to define $T \wedge N = N$, $F \vee N = N$, and $N' = N$.

Suppose that the statement "Flight 237 is on time" is true, the statement "Runway conditions are icy" is false, and the truth value of the statement "Flight 51 is on time" is unknown. Find the truth values of the following statements:

b. Runway conditions are not icy and flight 51 is on time.
c. Flight 51 is on time and flight 237 is not.
d. Flight 51 is not on time or runway conditions are not icy.

37. $(Y \to Z') \wedge (X' \to Y) \wedge [Y \to (X \to W)] \wedge (Y \to Z) \to (Y \to W)$
★ 38. $(A \wedge B)' \wedge (C' \wedge A)' \wedge (C \wedge B')' \to A'$
39. $(P \vee (Q \wedge R)) \wedge (R' \vee S) \wedge (S \to T') \to (T \to P)$

Using propositional logic, including the rules in Table 1.14, prove that each argument in Exercises 40–48 is valid. Use the statement letters shown.

40. If the program is efficient, it executes quickly. Either the program is efficient, or it has a bug. However, the program does not execute quickly. Therefore it has a bug.
E, Q, B
41. If Jane is more popular, then she will be elected. If Jane is more popular, then Craig will resign. Therefore if Jane is more popular, she will be elected and Craig will resign.
J, E, C
42. If chicken is on the menu, then don't order fish, but you should have either fish or salad. So if chicken is on the menu, have salad. *C, F, S*
43. The crop is good, but there is not enough water. If there is a lot of rain or not a lot of sun, then there is enough water. Therefore the crop is good and there is a lot of sun.
C, W, R, S
44. If the ad is successful, then the sales volume will go up. Either the ad is successful or the store will close. The sales volume will not go up. Therefore the store will close.
A, S, C
★ 45. Russia was a superior power, and either France was not strong or Napoleon made an error. Napoleon did not make an error, but if the army did not fail, then France was strong. Hence the army failed and Russia was a superior power. *R, F, N, A*
46. It is not the case that if electric rates go up, then usage will go down, nor is it true that either new power plants will be built or bills will not be late. Therefore usage will not go down and bills will be late. *R, U, P, B*
47. If Jose took the jewelry or Mrs. Krasov lied, then a crime was committed. Mr. Krasov was not in town. If a crime was committed, then Mr. Krasov was in town. Therefore Jose did not take the jewelry. *J, L, C, T*
48. If the birds are flying south and the leaves are turning, then it must be fall. Fall brings cold weather. The leaves are turning but the weather is not cold. Therefore the birds are not flying south. *B, L, F, C*

★ 49. a. Use a truth table to verify that $A \to (B \to C) \leftrightarrow (A \wedge B) \to C$ is a tautology.
b. Prove that $A \to (B \to C) \Leftrightarrow (A \wedge B) \to C$ by using a series of equivalences.
c. Explain how this equivalence justifies the deduction method that says:
to prove $P_1 \wedge P_2 \wedge \cdots \wedge P_n \to (R \to S)$, deduce S from $P_1, P_2, \ldots, P_n$, and R.
50. The argument of the defense attorney at the beginning of this chapter was

> If my client is guilty, then the knife was in the drawer. Either the knife was not in the drawer or Jason Pritchard saw the knife. If the knife was not there on October 10, it follows that Jason Pritchard didn't see the knife. Furthermore, if the knife was there on October 10, then the knife was in the drawer and also the hammer was in the barn. But we all know that the hammer was not in the barn. Therefore, ladies and gentlemen of the jury, my client is innocent.

Use propositional logic to prove that this is a valid argument.